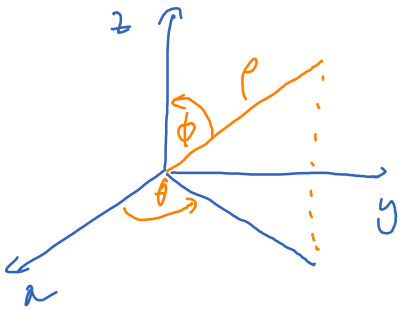
Lecture 29

Wednesday, March 16, 2022 9:48 AM

* Prayer
* Spiritual thought
* Spherical coords



$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$

$0 \le \theta \le 2\pi$,
$0 \le \phi \le \pi$.

Question: how to get ρ, ϕ, θ from x, y, z?

Ex $(x, y, z) = (0, 1, \sqrt{3})$

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{0^2+1^2+\sqrt{3}^2} = 2$$

$$\cos\phi = \frac{z}{\rho} = \frac{\sqrt{3}}{2} \rightsquigarrow \phi = \frac{\pi}{6}$$

$$\cos\theta = \frac{x}{\rho \sin\phi} = 0$$

$$\sin\theta = \frac{y}{\rho \sin\phi} = \frac{1}{2 \times \frac{1}{2}} = 1$$

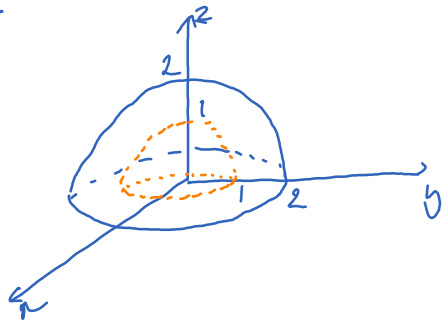
$$\Big\} \theta = \frac{\pi}{2}$$

Therefore, $(\rho, \theta, \phi) = (2, \frac{\pi}{2}, \frac{\pi}{6})$.

To use spherical coordinates to compute integrals, we need to know the Jacobian:

$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} = \rho^2 \sin\phi$$

Ex



$$\text{vol} = \iiint_E 1\, dV$$

$$(x,y,z) \to (\rho,\theta,\phi)$$

$$\left.\begin{array}{l} 1 \le \rho \le 2, \\ 0 \le \theta \le 2\pi, \\ 0 \le \phi \le \frac{\pi}{2} \end{array}\right\} E'$$

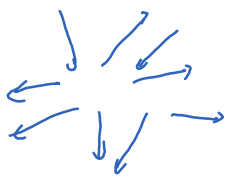
$$\iiint_E 1\, dV = \iiint_{E'} 1\, \rho^2 \sin\phi\, dV' = \int_0^{\pi/2}\int_0^{2\pi}\int_1^2 \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi = \cdots = \frac{14\pi}{3}$$

In Chapter 16, we will make sense of Integration by Parts for multivariable functions.

Vector fields: draw

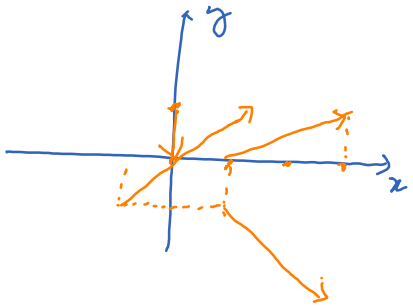
↳ a map of arrows

An example of vector field is the gradient vector field.



Ex: $F(x,y) = \langle 2x, x-y \rangle$

x	y	$F(x,y)$
0	1	$\langle 0,-1 \rangle$
1	0	$\langle 2,1 \rangle$
1	-1	$\langle 2,2 \rangle$
-1	1	$\langle -2,-2 \rangle$
2	0	$\langle 4,2 \rangle$



On Mathematica:

```
VectorPlot[{2x, x-y}, {x,-1,1}, {y,-1,1}]
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VectorPlot3D[{-y, x, xz}, {x,-5,5}, {y,-5 5}, {z,-5,5}]
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