Lecture 32

Thursday, March 24, 2022 11:14 PM

* Prayer
* Spiritual thought

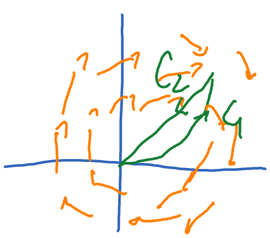
Line integral of a scalar function: $\int_C f(x,y,\dots)\underbrace{ds}_{|r'(t)|dt}$

Line integral of a vector field: $\int_C f(x,y,\dots)\cdot \underbrace{dr}_{r'(t)dt}$

orientation on the curve matters.

Ex $f(x,y) = (y,-x)$

C is made of two peaces:



$C_1: r_1(t) = (t, t^2) \quad 0 \le t \le 1$

$C_2: r_2(t) = (1-t, 1-t) \quad 0 \le t \le 1$

$$\int_C f\cdot dr = \int_{C_1} f\cdot dr + \int_{C_2} f\cdot dr$$

$$\int_{C_1} f\cdot dr = \int_C (y,-x)\cdot dr = \int_0^1 (t^2,-t)\cdot r'(t)\,dt = \int_0^1 (t^2,-t)\cdot(1,2t)\,dt$$

$$= \int_0^1 (t^2-2t^2)\,dt = -\frac{1}{3}$$

If F is "special" enough, one can expect that it is easy to evaluate

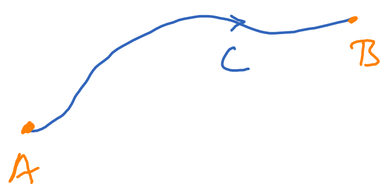
$$\int_C F \cdot ds.$$

- Calc I: $\int_a^b f(x)\,dx$

  If $f = g'$ then $\int_a^b f(x)\,dx = g(b) - g(a)$

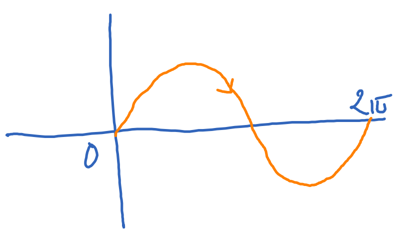
- Wish: F is the "derivative" of some function.

  If $F = \nabla\phi$ then $\int_C F \cdot dr = \int_C \nabla\phi \cdot dr = \phi(B) - \phi(A)$



If $F = \nabla\phi$ for some function ϕ then F is called a <u>conservative</u> vector field and ϕ is called the potential function of F.

<u>Ex</u>



$F(x,y) = (y, x)$

$\int_C F \cdot dr = ?$

Note: $F = \nabla\phi$ where $\phi(x,y) = xy$

$$\int_C F \cdot dr = \phi(2\pi, 0) - \phi(0,0) = 0.$$

* About notation:

$$\left.\begin{aligned} F &= (P, Q) \\ dr &= (dx, dy) \end{aligned}\right\} \quad F \cdot dr = P\,dx + Q\,dy$$