Lecture 6

Thursday, January 13, 2022 6:30 PM

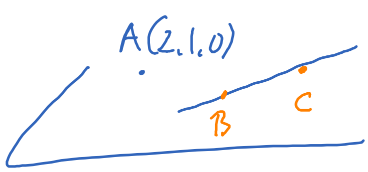
* Prayer

* Spiritual thought : go back to the basic

line $\begin{cases} \text{point} \\ \text{direction vector} \end{cases}$

plane $\begin{cases} \text{point} \\ \text{normal vector} \end{cases}$

Ex



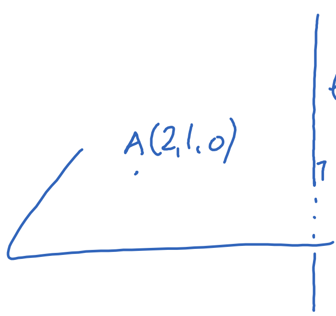
$l : \begin{cases} x = 1+t \\ y = 1-t \\ z = 2t \end{cases}$ write eq. of the plane

Take two points on the line .

$\left.\begin{array}{l} t = 0 \rightsquigarrow B(1,1,0) \\ t = 1 \rightsquigarrow C(2,0,2) \end{array}\right\} \begin{array}{l} \vec{AB} = \dots \\ \vec{AC} = \dots \end{array}$

normal vector of the plane : $\vec{AB} \times \vec{AC}$

Ex :



$l : \begin{cases} x = 1+t \\ y = 1-t \\ z = 2t \end{cases}$

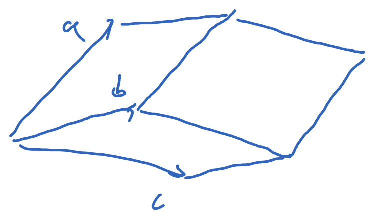
normal vector of the plane is a direction vector of the line :

$v = \langle 1, -1, 2 \rangle$

Triple product.

$$a\cdot(b\times c) = \det\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \det\begin{pmatrix} | & | & | \\ a & b & c \\ | & | & | \end{pmatrix}$$

Geometric meaning: $|a\cdot(b\times c)|$ = volume of the parallelepiped formed by a, b, c.



Triple product is used to check if 3 vectors lie on the same plane.

* Surfaces & curves

$\begin{cases} \text{cylinder} \\ \text{quadratic} \end{cases}$

1 equation $\longrightarrow$ surface

2 equations $\longrightarrow$ curves

[It is more convenient to describe curves using parametric equations]

Cylinder: described by an equation in which one of the variables x, y, z is missing. A translation or rotation of a cylinder is also called a cylinder.

$x = \sin y$

$x + z = 5$

$x^2 - y = y^2$

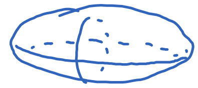
$x = y^2$: parabolic cylinder

$y^2 + z^2 = 4$: circular cylinder

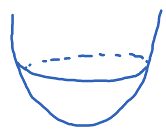
Quadratic surfaces

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz + J = 0$$

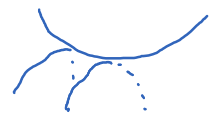
Ellipsoid



Paraboloid



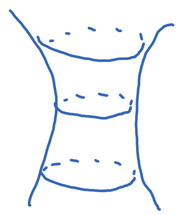
elliptic paraboloid



hyperbolic paraboloid

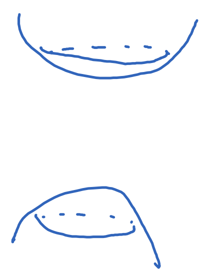
$z = x^2 - y^2$

Hyperboloid



one-sheet

$x^2 + y^2 - z^2 = 1$



two-sheet

$x^2 + y^2 - z^2 = -1$

Curves

$$r(t) = \langle x(t), y(t), z(t) \rangle \qquad a \le t \le b$$

Ex: $r(t) = \langle \cos 5t, \sin 5t, t \rangle$

$$r(t) = \langle \sqrt{1-t^2} \cos 5t, \sqrt{1-t^2} \sin 5t, t \rangle$$

Limit: $\lim\limits_{t \to t_0} r(t) = \langle \lim\limits_{t \to t_0} x(t), \lim\limits_{t \to t_0} y(t), \lim\limits_{t \to t_0} z(t) \rangle$