# Line and surface integral

The command **Integrate** in Mathematica not only can compute single, double, triple, iterated integrals, but also can compute line integral and surface integrals. The main step is to declare the curve or surface to be integrated over using the command **ImplicitRegion**. Below are two examples.

1. Line integral
    Integrate the function $f(x, y) = x + y$ over the right half of the ellipse $x^2 + 4y^2 = 1$. First,
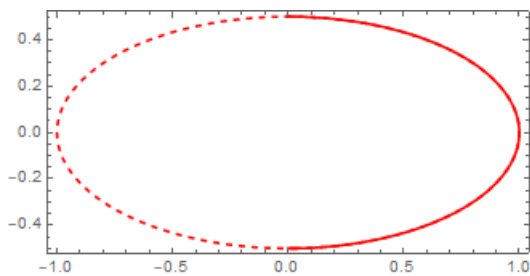
    we need to describe the curve using (in)equalities. These are $x^2 + 4y^2 = 1$ and $x \geq 0$.

    ```
    curve = ImplicitRegion[4 x^2 + y^2 == 1 && x >= 0, {x, y}]
    ```

    The part {x,y} in the above command is to suggest that the curve is a collection of pairs (x, y) satisfying the one equality and one inequality as indicated. Next, we integrate the function $f(x, y)$ over the curve.

    ```
    Integrate[x + y, {x, y} \[Element] curve]
    ```

2. Surface integral
    Find the area of the portion of the surface $z = 4 - 2x^2 + y$ that lies above the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$. First, we need to describe the surface using (in)equalities. The surface is a collection of points (x, y, z) such that $z = 4 - 2x^2 + y$, $0 \leq x \leq 1$, and $0 \leq y \leq x$.

    ```
    surface =  ImplicitRegion[
      z == 4 - 2 x^2 + y && 0 <= x <= 1 && 0 <= y <= x, {x, y, z}]
    ```

    Next, the area of the surface is obtained by integrating the constant function 1 over the surface itself.

    ```
    Integrate[1, {x, y, z} \[Element] surface]
    ```