Midterm II: Some problems for review

The exam is 2 hours long and taken at the Testing Center between March 9 and March 11. It is a closed book exam, covering Chapter 14 and Sections 15.1, 15.2, 15.3, 15.6. No calculators are allowed.

You should review methods to solve following types of problems. Also, take a look at the practice exam posted on Learning Suite. *The quickest way to review is perhaps by reading examples given in lectures or the textbook. It is always a good idea to study for the exam with someone.*

- Find limit of a function.
- Find linear approximation of a function around a given point.
- Find partial derivatives of a function.
- Write the total differential of a function.
- Write equation of a tangent plane to a surface at a given point.
- Find local/absolute min/max and saddle points of a function with or without constraints.
- Evaluate double/triple integral.
- Find volume/mass of a solid.

1) Compute $\iint_D \frac{y}{1+x^2} dA$ where D is the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$.

2) Compute $\iiint_E z dV$ where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

3) Find the maximum and minimum values of $f(x, y) = x^2 + y^2 + 4x - 4y$ in the disk $x^2 + y^2 \leq 9$.

4) Write the equation of the tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at point $(1, -2, 1)$.

5) The rate of change of function $f(x, y) = xy + y^2$ in the direction of vector $\langle 0, 1 \rangle$ at point $(2, 1)$ is ______. At this point, the function increases the fastest in the direction of (unit) vector ______.

6) A function $f(x, y)$ satisfying $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = f(x_0, y_0)$ is said to be __________ at (x_0, y_0).

7) Where is the function $f(x, y) = \frac{e^x+e^y}{e^{xy}-1}$ continuous?

8) Along a level set of a function, the rate of change of the function is ________.

9) The graph of $f(x, y)$ is _______________ of $g(x, y, z) = z - f(x, y)$.

10) Let $u = \ln(1 + se^t)$. Express the total differential du in terms of ds and dt.

11) By Clairaut's Theorem, a smooth function $f(x, y)$ has at most ________ different partial derivatives of third order.

12) Let $f(x, y) = ax(1 + y) + by$. If $\nabla f(1, 1) = \langle 2, 1 \rangle$ then $a =$_____ and $b =$_____.

13) Find linear approximation of $f(x, y) = x^3 - 2xy^2$ around $(1, 1)$.

14) What is the difference between local minimum/maximum and saddle point? How do you distinguish them using Second Derivative test?

15) A function $f(x, y)$ has at most two critical points. True or false?

16) The absolute maximum over $\mathbb{R}^2$ of a function $f(x, y)$, if exists, must be attained at a critical point. True or false?

17) If $f(x, y) \to L$ as $(x, y) \to (a, b)$ along every straight line through (a, b), then $\lim_{(x,y)\to(a,b)} f(x, y) = L$. True or false?

18) $\lim_{(x,y)\to(1,1)} \frac{2xy^2}{x^2+y^2} =$ ______ (or write DNE if the limit doesn't exist.)

19) $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^2} =$ ______

20) $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} =$ ______

Solution keys:

1) $\frac{1}{4} \ln 2$

2) $\frac{8\pi}{3}$

3) min $= -8$, max $= 9 + 12\sqrt{2}$

4) $z = 8x + 4y + 1$

5) 4 and $\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \rangle$

6) continuous

7) Everywhere in $\mathbb{R}^2$ except for the x-axis and the y-axis

8) zero

9) the 0-level set

10) $du = \frac{e^t}{1+se^t} ds + \frac{se^t}{1+se^t} dt$

11) 4

12) $a = 1$ and $b = 0$

13) $f(x, y) \approx -1 + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) = x - 4y + 2$

14)

15) False. Can you give an example?

16) True

17) False. Can you give an example?

18) 1

19) 0

20) DNE