Optimization under constraints

Our purpose is to visualize the optimization of the function $f(x, y) = x^3+y^3$ under the constraint $x^2 + y^2 = 1$ using Mathematica. First, we plot a large number of level sets of f, say 200 level sets (Figure 1).

```
p1 = ContourPlot[x^3 + y^3, {x, -2, 2}, {y, -2, 2}, Contours -> 200]
```

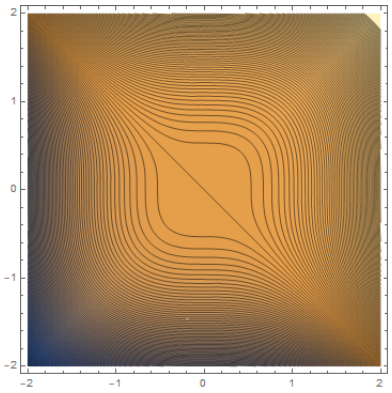

Figure 1

You can hover your mouse on each level curve to see the value of f on that curve. Next, we plot the curve $x^2 + y^2 = 1$. This is the unit circle centered at the origin on the xy-plane (Figure 2).

```
p2 = ContourPlot[x^2 + y^2 == 1, {x, -2, 2}, {y, -2, 2},
ContourStyle -> Red]
```

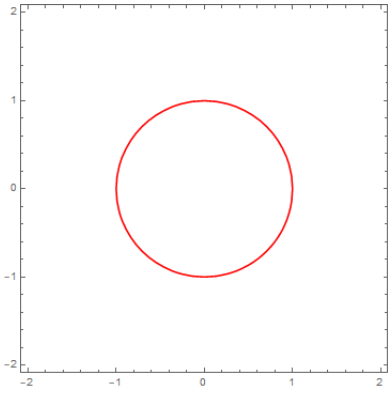

Figure 2

Next, we put two figures on the same plot (Figure 3).

```
Show[p1,p2]
```

Now look at the level curves of f that are tangent to the red unit circle. You can observe that there are four such curves. These are the four candidates for min/max (Figure 4). Hovering the mouse over these four level curves, you will see which curve gives minimum/maximum value.

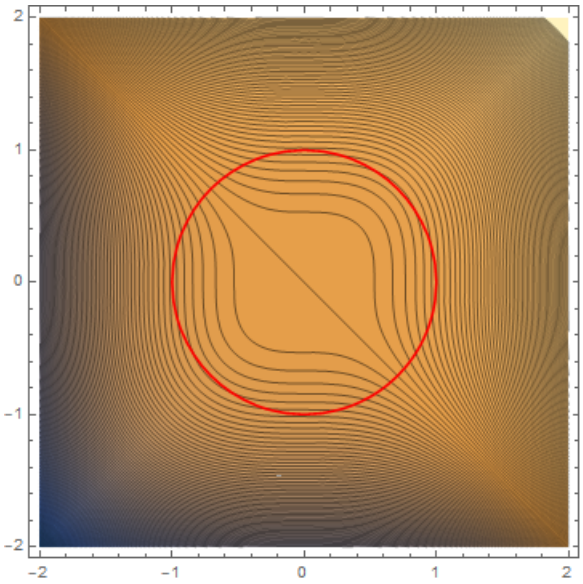

Figure 3

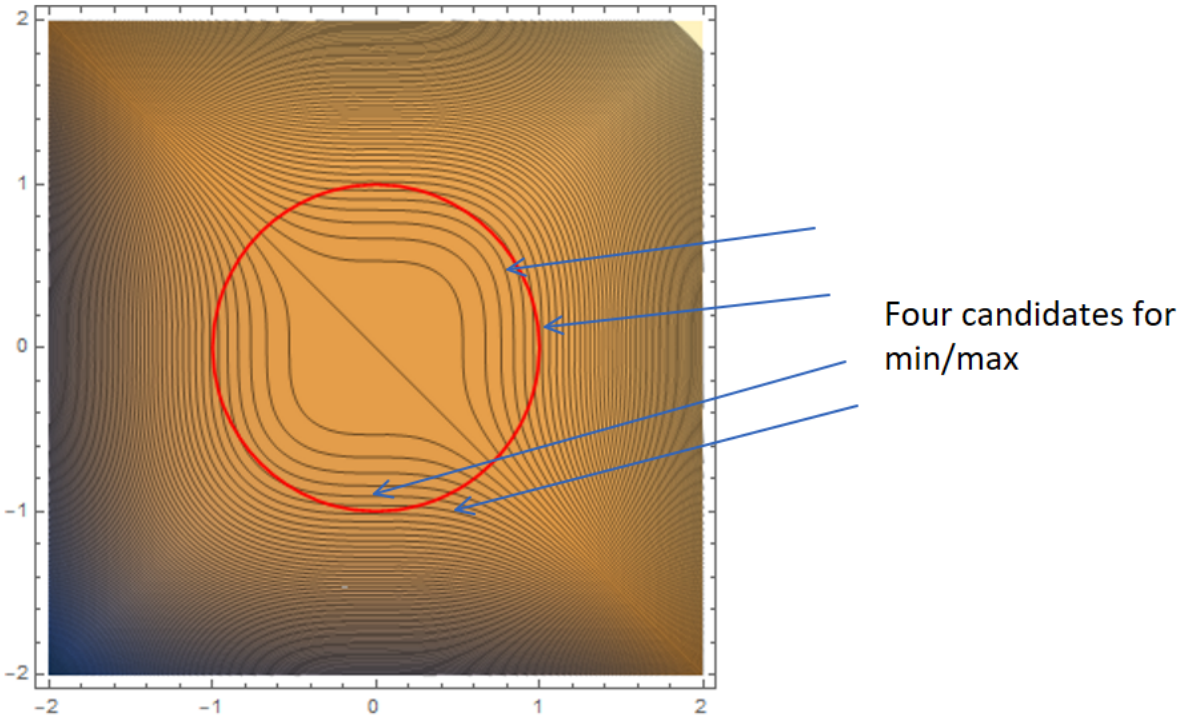

Figure 4