Worksheet
11/28/2018

You may use calculator to compute RREF **only**. For Problem 1, you should do row reduction by hand.

1. Find the inverse of matrix using row reduction method.

$$\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & -17 \\ 3 & 1 & -1 \end{bmatrix}$$

2. Find all values c such that the following matrix is invertible

$$\begin{bmatrix} 1 & c & 0 \\ c & 1 & 0 \\ 0 & 1 & c \end{bmatrix}$$

3. Let $S = \{v_1, v_2, v_3\}$ where

$$\begin{aligned} v_1 &= (1, 1, 1) \\ v_2 &= (1, 2, 3) \\ v_3 &= (1, 1, 2) \end{aligned}$$

(a) Check if S is a basis of $\mathbb{R}^3$.

(b) Let $v = (2, 1, 3)$. Find the coordinate of v in basis S.

4. Let v_1, v_2, v_3 be given as in the previous problem. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map satisfying:

$$\begin{aligned} f(v_1) &= (1, 3) \\ f(v_2) &= (-1, 0) \\ f(v_3) &= (0, 5) \end{aligned}$$

(a) Find the matrices representing f in the standard basis.

(b) Determine the kernel of f, a basis and the dimension.

(c) Determine the range of f, a basis and the dimension.

5. Diagonalize the matrix

$$\begin{bmatrix} -2 & 12 \\ -1 & 5 \end{bmatrix}$$

6. Solve the following system of differential equations

$$\begin{cases} x_1' &= -2x_1 + 12x_2 \\ x_2' &= -x_1 + 5x_2 \end{cases}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = -1$.