

Worksheet
11/30/2018

You may use calculator to compute RREF **only**.

1. Let

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

- (a) Find A^{-1} (if exists).
- (b) Express A as a product of elementary matrices.

2. Find the following determinant by row reduction method. Show each row operation step.

$$\begin{vmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 1 & 3 \\ 0 & 5 & 0 & -6 \end{vmatrix}$$

3. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $g : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be linear maps such that

$$\begin{aligned} f(1, 2) &= (1, 4) & g(5, 1) &= (-1, 1, 0) \\ f(2, 3) &= (-1, 3) & g(4, 1) &= (2, 3, -1) \end{aligned}$$

- (a) Find the matrix representing f in standard basis. What is $f(12, 7)$?
- (b) Find the matrix representing g in standard basis.
- (c) Find the matrix representing $g \circ f$ in standard basis.

4. Let V be a subspace of \mathbb{R}^5 spanned by the vectors

$$v_1 = (1, 1, 1, 4, 5)$$

$$v_2 = (1, 2, 3, 2, 1)$$

$$v_3 = (-1, -3, -5, 0, 3)$$

(a) Find a basis of V . What is the dimension of V ?

(b) Supplement more vectors to this basis to obtain a basis of \mathbb{R}^5 .

5. Determine all values of c such that the map $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$,

$$f(x, y, z) = (x + y + cz, cx + cy + z)$$

is surjective.