Homework 1: Answers to selected problems.

1 g) $\begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (you can use the command rref in Matlab to double check).

2 b) Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Therefore, $x = 2$, $y = -1$, $z = 1$.

g) Augmented matrix:

$$\left[\begin{array}{cc|c} 1 & -m & 1 \\ 3 & -7 & 5 \end{array}\right] \xrightarrow{R_2 = R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & -m & 1 \\ 0 & -7+3m & 2 \end{array}\right]$$

- If $-7+3m = 0$ (or $m = 7/3$) then the system becomes

$$\left[\begin{array}{cc|c} 1 & -7/3 & 1 \\ 0 & 0 & 2 \end{array}\right],$$ which has no solutions because of the last row.

- If $-7+3m \neq 0$ (that is, $m \neq 7/3$),

$$\left[\begin{array}{cc|c} 1 & -m & 1 \\ 0 & -7+3m & 2 \end{array}\right] \xrightarrow{R_2 = \frac{R_2}{-7+3m}} \left[\begin{array}{cc|c} 1 & -m & 1 \\ 0 & 1 & \frac{2}{-7+3m} \end{array}\right]$$

$$\xrightarrow{R_1 = R_1 + mR_2} \left[\begin{array}{cc|c} 1 & 0 & 1+\frac{2m}{-7+3m} \\ 0 & 1 & \frac{2}{-7+3m} \end{array}\right]$$

In this case, the system has a unique solution:

$$x = 1 + \frac{2m}{-7+3m} = \frac{-7+5m}{-7+3m}$$

$$y = \frac{2}{-7+3m}$$