Homework Set 2

Due 10/5/2018

1. Find the rank of the following matrices

(a)

$$\begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ -3 & -10 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 4 & -2 & 6 \\ 3 & 6 & -6 & 19 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 4 \\ 8 & 2 & 1 \\ 3 & 5 & 2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 4 & 5 & 2 & 3 \\ 1 & 8 & 2 & 3 \\ 2 & 3 & 4 & 2 \\ 8 & 2 & 5 & 3 \end{bmatrix}$$

2. Let

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Determine all matrices B of size 2×2 such that $BA = 0$. In other words, find all a, b, c, d satisfying

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Express matrix B in the form $B = \alpha_1 B_1 + \alpha_2 B_2$ where α_1, α_2 are arbitrary real numbers, and B_1, B_2 are constant matrices (i.e. each entry is a constant).

3. Find the product AB and BA. If one of them does not exist, explain why.

(a)

$$A = \begin{bmatrix} 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 3 \\ 1 & 4 \\ 2 & 2 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 3 & 8 & -2 \\ 4 & 1 & 0 \\ 2 & 3 & 8 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

4. Let A be a 3×3 matrix. Suppose we want to do three elementary row operations to A in the following order.

(1) $R_2 = R_2 + 4R_1$

(2) $R_1 \leftrightarrow R_3$

(3) $R_3 = 5R_3$

Denote by B the resulting matrix. Determine matrix E such that $EA = B$.

5. Use Gauss elimination method to find inverse of matrix A. If the inverse does not exist, explain why.

(a)

$$A = \begin{bmatrix} 2 & 1 \\ -4 & -3 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$$

6. Determine all values of c such that the following matrix is NOT invertible.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 4 & c \end{bmatrix}$$