

Problem 6, part (e):

$$\left| \begin{array}{cccc|l} 2 & 1 & 0 & 4 & \\ 1 & 2 & 1 & 4 & R_4 = R_4 - R_1 \\ 0 & 3 & 2 & 2 & R_1 = R_1 - 2R_2 \\ 2 & 1 & 3 & 3 & R_1 \leftrightarrow R_2 \end{array} \right| - \left| \begin{array}{cccc|l} 1 & 2 & 1 & 4 & \\ 0 & -3 & -2 & -4 & \\ 0 & 3 & 2 & 2 & \\ 0 & 0 & 3 & -1 & \end{array} \right|$$

$$\begin{array}{l} \underline{R_3 = R_3 + R_2} \\ \underline{R_3 \leftrightarrow R_4} \end{array} \left| \begin{array}{cccc|l} 1 & 2 & 1 & 4 & \\ 0 & -3 & -2 & -4 & \\ 0 & 0 & 0 & -2 & \\ 0 & 0 & 3 & -1 & \end{array} \right| \left| \begin{array}{cccc|l} 1 & 2 & 1 & 4 & \\ 0 & -3 & -2 & -4 & \\ 0 & 0 & 3 & -1 & \\ 0 & 0 & 0 & -2 & \end{array} \right|$$

$$= 1 \cdot (-3) \cdot 3 \cdot (-2)$$

$$= 18$$

Problem 4:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{R_3 = R_3 - R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 = R_3 - R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{array}{l} R_1 = R_1 + R_3 \\ R_2 = R_2 + R_3 \\ R_3 = -R_3 \end{array} \xrightarrow{\quad} I_3$$

The reverse chain of elementary row operations from I_3 to A is

$$(1) \quad R_3 = -R_3 \dots \dots \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$(2) \quad R_2 = R_2 - R_3 \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(3) \quad R_1 = R_1 - R_3 \quad E_3 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(4) \quad R_3 = R_3 + R_2$$

$$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$(5) \quad R_3 = R_3 + R_1$$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

We see that

$$I_3 \xrightarrow{(1)} E_1 \xrightarrow{(2)} E_2 E_1 \xrightarrow{(3)} E_3 E_2 E_1 \xrightarrow{(4)} E_4 E_3 E_2 E_1 \xrightarrow{(5)} \underbrace{E_5 E_4 E_3 E_2 E_1}_A$$

Note: this factorization ($A = E_5 \dots E_2 E_1$) is not unique. You can have a different way to factor A depending on the elementary row operations you use.