

$$2d) \quad v_1 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

* Check if v_1, v_2, v_3 are linearly independent:

The equation $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ is equivalent to the following system:

$$\begin{bmatrix} 0 & 3 & 1 \\ 0 & 3 & 1 \\ 2 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix} \xrightarrow{\substack{R_4 = R_4 - R_3 \\ R_3 = R_3/2 \\ R_1 \leftrightarrow R_3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{\substack{R_3 = R_3 - R_2 \\ R_2 = R_2/3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/3 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{\substack{R_2 = R_2 + 1/3 R_4 \\ R_4 = -R_4 \\ R_4 \leftrightarrow R_3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

All columns are pivot. Thus, $c_1 = c_2 = c_3 = 0$ is the unique sol. to the system. Vectors v_1, v_2, v_3 are linearly independent.

$$4d) \quad V = \{ (x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 x_3 = 0 \}$$

this term indicates that

V is probably not a subspace.

$$\text{Check: } v = (0, 1, 0) \in V$$

$$w = (1, 0, -2) \in V$$

But $v + w = (1, 1, -2) \notin V$. Thus V is not closed under addition.

Conclusion: V is not a subspace of \mathbb{R}^3 .