Homework Set 4

Due 10/19/2018

1. Find the determinant of the following matrices. Show your work.

(a)
$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

2. To each following set of vectors, do the following:

(1) Check if they are linearly independent.

(2) If they are linearly dependent, write one vector as a linear combination of the others.

(3) Find a basis for the space spanned by them.

(a) $v_1 = (1, 3, -1)$, $v_2 = (3, 7, -7)$, $v_3 = (1, 2, -3)$.

(b) $v_1 = (2, 1, 3)$, $v_2 = (1, 0, 1)$, $v_3 = (0, 2, -1)$, $v_4 = (4, 2, 1)$.

(c) $v_1 = (3, 8, 7, -3)$, $v_2 = (1, 5, 3, -1)$, $v_3 = (2, -1, 2, 6)$, $v_4 = (1, 4, 0, 3)$.

(d) $v_1 = (0, 0, 2, 2)$, $v_2 = (3, 3, 0, 0)$, $v_3 = (1, 1, 0, -1)$.

3. Find a basis for the subspace $\{x \in \mathbb{R}^4 : \ Ax = 0\}$ of $\mathbb{R}^4$ where

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & 2 & 0 \\ 1 & 4 & 8 & 2 \end{bmatrix}$$

This is called the *null space* of matrix A.

4. Check if each following set is a subspace of $\mathbb{R}^n$.

(a) $V = \{x = (x_1, x_2) : \ x_1 + 2x_2 = 0\}$, a line through the origin in $\mathbb{R}^2$.

(b) $V = \{x = (x_1, x_2) : \ x_1 + x_2 = 1\}$, a line not passing through the origin in $\mathbb{R}^2$.

(c) $V = \{x = (x_1, x_2) : \ x_2 = x_1^2\}$, a parabola in $\mathbb{R}^2$.

(d) $V = \{x = (x_1, x_2, x_3) : \ x_1 + x_2 x_3 = 0\}$ as a subset in $\mathbb{R}^3$.