

2) b. v_1, v_2, v_3, v_4 are vectors in \mathbb{R}^3 . The number of vectors (4) exceeds the dimension of \mathbb{R}^3 (3). Thus, these vectors are linearly dependent. Express one vector as a linear comb. of the others:

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$$

This eq. is equivalent to the following system:

$$\begin{array}{cccc|c} \begin{array}{c} \downarrow c_2 \\ 1 \\ 2 \\ -1 \end{array} & \begin{array}{c} \downarrow c_2 \\ 2 \\ 1 \\ 3 \end{array} & \begin{array}{c} \downarrow c_3 \\ -1 \\ 0 \\ 4 \end{array} & \begin{array}{c} \downarrow c_4 \\ 0 \\ 3 \\ 1 \end{array} & \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \end{array} \xrightarrow{\text{RREF}} \begin{array}{cccc|c} 1 & 0 & 0 & 32/19 & 0 \\ 0 & 1 & 0 & -7/19 & 0 \\ 0 & 0 & 1 & 18/19 & 0 \end{array}$$

↑
nonpivot col.

$$\begin{cases} c_4 = t, \\ c_3 = -\frac{18}{19}t \\ c_2 = \frac{7}{19}t \\ c_1 = -\frac{32}{19}t \end{cases} \quad \text{Pick } t = 19: \quad \begin{cases} c_4 = 19 \\ c_3 = -18 \\ c_2 = 7 \\ c_1 = -32 \end{cases}$$

We have $-32v_1 + 7v_2 - 18v_3 + 19v_4 = 0$

This implies: $v_4 = \frac{32}{19}v_1 - \frac{7}{19}v_2 + \frac{18}{19}v_3$

The space spanned by v_1, v_2, v_3, v_4 has basis $\{v_1, v_2, v_3\}$ because the first 3 columns in the RREF are pivot. This space has dimension 3.

4) $A \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 5/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

↑ ↑
nonpivot cols.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$x_4 = t$$

$$x_3 = s$$

$$x_2 = -\frac{5}{2}s - \frac{1}{2}t, \quad x_1 = 2s$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2s \\ -5/2s - 1/2t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} 2 \\ -5/2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}$$

The null space of A is the space spanned by vectors

$$v_1 = \begin{bmatrix} 2 \\ -5/2 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad v_2 = \begin{bmatrix} 0 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}$$

These vectors are linearly independent, thus form a basis for $N(A)$.
 $\dim N(A) = 2$.