Homework Set 5

Due 10/26/2018

1. Determine if vector $b = (-2, 3, 6)$ is a linear combination of the following vectors:

$$v_1 = (3, 1, -1),\ v_2 = (-1, 2, -3),\ v_3 = (1, -1, 2),\ v_4 = (2, 1, -4).$$

   If yes, write b as a linear combination of these vectors.

2. To each following set of vectors, do the following:

   (1) Check if they are linearly independent.

   (2) If they are linearly dependent, write one vector as a linear combination of the others.

   (3) Find a basis for the space spanned by them. What is the dimension?

   (a) $v_1 = (2, 3, 0)$, $v_2 = (1, 2, -1)$, $v_3 = (0, 4, 3)$.

   (b) $v_1 = (1, 2, -1)$, $v_2 = (2, 1, 3)$, $v_3 = (-1, 0, 4)$, $v_4 = (0, 3, 1)$.

   (c) $v_1 = (-1, 0, 4, 3)$, $v_2 = (2, 0, -3, 2)$, $v_3 = (0, 1, -1, 3)$, $v_4 = (4, -2, 1, 6)$.

   (d) $v_1 = (0, 2, 1, -1, 1)$, $v_2 = (1, 0, 3, 2, 0)$, $v_3 = (-1, 1, 2, 3, 2)$.

3. Supplement additional vectors to the set $\{v_1, v_2, v_3\}$ in Part (d) of Problem 2 to obtain a basis of $\mathbb{R}^5$.

4. Determine a basis and the dimension for the subspace $\{x \in \mathbb{R}^4 : \ Ax = 0\}$ of $\mathbb{R}^4$ where

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & 2 & 0 \\ 1 & 4 & 8 & 2 \end{bmatrix}$$

   This space is called the *null space* of matrix A. Its dimension is called the *nullity* of A.