# Homework Set 6

Due 11/02/2018

**Note:** In this homework set, you are allowed to use calculator or Matlab to compute RREF. However, for other tasks such as finding inverse or computing determinant, you need to write in detail every step leading up to the point where you compute RREF.

1. Check if each following set is a subspace. If it is, explain why (by verifying the 3 criteria). If it is not, show how one of these criteria is violated.

   (a) $V = \{(x, y, z) : \ x^2 + yz = 0\}$ as a subset of $\mathbb{R}^3$.

   (b) $V = \{(x, y, z) : \ x + 2y - z = 0\}$ as a subset of $\mathbb{R}^3$.

   (c) $V = \{(x, y, z) : \ xyz = 0\}$ as a subset of $\mathbb{R}^3$.

2. Check if each following map is a linear map. If it is, explain why (by verifying the 2 criteria). If it is not, show how one of these criteria is violated.

   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = x^2 + y$.

   (b) $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x, y, z) = x + 2y + z$.

   (c) $f : \mathbb{R}^3 \to \mathbb{R}^2$, $f(x, y, z) = (x + y, \ x + yz)$.

3. Consider the linear maps

$$\begin{aligned}
f &: \mathbb{R}^4 \to \mathbb{R}^3, \quad f(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4, \ x_2 - x_3, \ -2x_1 - 4x_3 + x_4) \\
g &: \mathbb{R}^3 \to \mathbb{R}^3, \quad g(x_1, x_2, x_3) = (x_1 + x_2 + x_3, \ 6x_1 + 5x_2 + 4x_3, \ 13x_1 + 10x_2 + 8x_3) \\
h &: \mathbb{R}^3 \to \mathbb{R}^3, \quad h(x_1, x_2, x_3) = (x_2, x_3, x_1).
\end{aligned}$$

   (a) Find the matrices representing $f$, $g$, $h$, $2f$, $g \circ f$, and $g + h$.

   (b) Find the inverse of $g$ (if exists). In other words, write an explicit expression for $g^{-1}(x_1, x_2, x_3)$.

4. Let $f$ be the linear map in Problem 3.

   (a) Find a basis and the dimension of $\ker(f)$.

   (b) Find a basis and the dimension of $\mathrm{range}(f)$.

5. Determine all values of $c$ such that the following linear map is bijective:

$$f : \mathbb{R}^3 \to \mathbb{R}^3, \quad f(x, y, z) = (x + 2y - z, \ x + (c + 2)y - z, \ x + 2y + cz).$$

   Hint: Bijective means being both injective (kernel equal to $\{0\}$) and surjective (range equal to $\mathbb{R}^3$).