

Solution to Prob. 4 of Homework 7

Let A of size 2×3 be the matrix representing f .

$$f(2,3,1) = (1,0) \Rightarrow A \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$f(1,0,1) = (2,-1) \Rightarrow A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$f(-1,-2,0) = (-1,1) \Rightarrow A \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Now combine these three equations together:

$$A \underbrace{\begin{bmatrix} 2 & 1 & -1 \\ 3 & 0 & -2 \\ 1 & 1 & 0 \end{bmatrix}}_P = \underbrace{\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}}_B$$

Then multiply both side by P^{-1} to the right:

$$A = B P^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 2 & -1 & -1 \\ -3 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -2 & -3 \\ -5 & 2 & 4 \end{bmatrix}$$

This is the matrix representing f .

$$f(3,4,5) = A \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 & -2 & -3 \\ -5 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -8 \\ 13 \end{bmatrix}$$