Homework Set 8

Due 11/21/2018

Note: In this homework set, you are allowed to use calculator or Matlab to compute RREF. For other tasks, you need to write in detail every step leading up to the point where you compute RREF.

1. Consider a linear map $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x,\, y,\, z) = (2x - y + z,\, y - 2z,\, x + z)$ and vectors

$$\begin{aligned} v_1 &= (0,\, 1,\, -3) \\ v_2 &= (-3,\, -4,\, 4) \\ v_3 &= (-2,\, -2,\, 1) \\ a &= (1,\, 2,\, 3) \\ b &= (3,\, 1,\, 4) \end{aligned}$$

   (a) Check if $S = \{v_1,\, v_2,\, v_3\}$ is a basis of $\mathbb{R}^3$.

   (b) Find the coordinates of vectors a and b in basis S. In other words, find $[a]_S$ and $[b]_S$.

   (c) Find the coordinates of vectors $f(a)$ and $f(b)$ in basis S. In other words, find $[f(a)]_S$ and $[f(b)]_S$.

   (d) Find matrix representing f in basis S. In other words, find $[f]_S$.

   (e) Check the validity of the formula $[f(v)]_S = [f]_S[v]_S$ when $v = a$ and $v = b$.

2. Consider the plane (P): $2x - y + 3z = 0$ in the 3-dimensional space. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be the projection onto this plane. In other words, f maps any point in the space to its projection on the plane.

   (a) Pick two linearly independent vectors lying on the plane and name them v_1 and v_2. Determine $f(v_1)$ and $f(v_2)$.

   (b) Pick a nonzero vector in the direction perpendicular to the plane and name it v_3. Determine $f(v_3)$.

   (c) This way of selecting $v_1,\, v_2,\, v_3$ guarantees that $S = \{v_1,\, v_2,\, v_3\}$ is a basis of $\mathbb{R}^3$. From the results in Part (a) and (b), determine $[f]_S$.

   (d) Determine the matrix representing f in the standard basis. Then write an explicit formula for $f(x, y, z)$.

3. Determine the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

4. Determine the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

5. In the 3-dimensional space, the rotation by angle θ about the z-axis is a linear map represented by matrix

$$A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For $\theta = 90^\circ$ (or equivalently $\pi/2$ radians), find the eigenvalues and corresponding eigenvectors of this rotation. You are expected to get complex-valued eigenvalues and eigenvectors.