Homework Set 9
Due 11/28/2018

Note: In this homework set, you are allowed to use calculator or Matlab to compute RREF. For other tasks, you need to write in detail every step leading up to the point where you compute RREF.

1. For each of the following matrices, find the eigenvalues and eigenvectors (either real-valued or complex-valued). Is the matrix diagonalizable? If yes, diagonalize it. If no, explain why.

   (a)
   $$\begin{bmatrix} 0 & 0 & -3 \\ -5 & 5 & 1 \\ 2 & -2 & -1 \end{bmatrix}$$

   (c)
   $$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -3 \\ -2 & 1 & 5 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 3 & 6 & 4 \end{bmatrix}$$

   (d)
   $$\begin{bmatrix} 1 & 4 & -2 \\ 0 & 0 & 1 \\ 0 & -5 & 4 \end{bmatrix}$$

2. Let
$$A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$$

   Find an explicit formula for A^n in terms of n by matrix diagonalization.

3. Let $x_0, x_1, x_2, x_3, \ldots$ be a sequence defined recursively as follows:

$$
\begin{aligned}
x_0 &= 0, \\
x_1 &= 1, \\
x_{n+1} &= 2x_n + 3x_{n-1}.
\end{aligned}
$$

   (a) Write the first 6 terms of this sequence.

   (b) Put $y_n = \begin{bmatrix} x_{n+1} \\ x_n \end{bmatrix}$. Find matrix A such that $y_n = Ay_{n-1}$.

   (c) Write a formula for y_n that involves only n, A, and y_0.

   (d) Write an explicit formula for x_n in terms of n.