Lab 1

Due 10/8/2018

I Instruction.

In this lab, we will use Matlab to (1) reduce matrices to reduced row echelon form (RREF) and (2) solve systems of linear equations. The first part is for practice. You don't need to include this part in your report. The second part (two exercises) is what you write a report on. You may want to quickly review Lab 0 for some basic commands. *A good length for this report is 1 - 2 pages.*

II Practice.

Practice 1: Gauss elimination.

Consider matrix

$$A = \begin{bmatrix} 2 & -3 & -1 \\ 1 & 4 & 5 \\ 0 & 1 & 3 \end{bmatrix}$$

To enter this matrix in Matlab, simply type

```
A = [2 -3 -1;1 4 5;0 1 3]
```

(see also Lab 0). Let us transform A into RREF by elementary row operations. In the process, A is changed after each step. You may want to store the original matrix so that you refer to it when need to:

```
B = A
```

We exchange the first and second row of A by implementing:

```
a = A(1,:)
A(1,:) = A(2,:)
A(2,:) = a
```

The resulting matrix is

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & -3 & -1 \\ 0 & 1 & 3 \end{bmatrix}$$

Then subtract two times the first row from the second row:

```
A(2,:) = A(2,:) - 2*A(1,:)
```

The resulting matrix is

$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & -11 & -11 \\ 0 & 1 & 3 \end{bmatrix}$$

Then divide the second row by -11:

```
A(2,:) = A(2,:)/(-11)
```

The resulting matrix is

$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

You can easily continue the rest of the procedure. At some point, you may see at star (*) at some coefficients. It occurs when the number is too small (and too long) for Matlab to render, for example

0.0000000001. In most cases, this is a machine error (error due to truncation, accumulated over many steps). The accurate number should be 0. If a star shows up at position (2,3), for example, at some step, you can assign it to be 0 before moving on to the next step:

```
A(2,3) = 0
```

After finishing the procedure, you can double check your result with the command:

```
rref(B)
```

Practice 2: Dealing with "ugly" numbers.
The numbers sin(1), π, tan(3), $\sqrt{2}$, ln(4) can be entered in Matlab as follows.

```
sin(1)
pi
tan(3)
sqrt(2)
exp(2)
log(4)
```

Now consider the matrix

$$A = \begin{bmatrix} \sin(1) & \pi & \tan(3) \\ \sqrt{2} & e^2 & \ln(4) \end{bmatrix}$$

Matlab will give you an unfriendly output as follows:

$$\begin{bmatrix} 0.8415 & 3.1416 & -0.1425 \\ 1.4142 & 7.3891 & 1.3863 \end{bmatrix}$$

It would be inconvenient to type decimal numbers such as A(1,:) = A(1,:)/0.8415. A simple trick to avoid this practice is to refer to the number 0.8415 by coordinate:

```
A(1,:) = A(1,:)/A(1,1)
```

In this way, you can continue to transform A into RREF without worrying about typing long numbers.

Practice 3. Rendering in fractions.
Consider matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \end{bmatrix}$$

You can get the RREF of this matrix directly by the command:

```
rref(A)
```

which gives you

$$\begin{bmatrix} 1.0000 & 0 & -0.3333 \\ 0 & 1.0000 & 1.6667 \end{bmatrix}$$

If you wish to see the result in fraction format, use the command:

```
format rational
rref(A)
```

which gives

$$\begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \end{bmatrix}$$

From this time forth, Matlab will try to render numbers in fractional forms (without you typing "format rational" again). If you want to switch back to decimal form, use the command:

```
format short
```

or "format long" depending on how many digits after the decimal point you want.

III Exercises.

1. Use Gauss elimination as in Practice 1 and 2 to transform the following matrix to RREF:

$$A = \begin{bmatrix} 2 & \pi \\ \sqrt{2} & \ln(2) \\ 0 & -1 \end{bmatrix}$$

   Make sure to write the explanation, command and output for each step. Then use rref command to double check your result.

2. Balance the following chemical equation

$$\mathrm{KMnO_4} + \mathrm{HCl} \rightarrow \mathrm{KCl} + \mathrm{MnCl_2} + \mathrm{H_2O} + \mathrm{Cl_2}$$

   In other words, find intergers x_1, x_2,..., x_6 such that

$$x_1\mathrm{KMnO_4} + x_2\mathrm{HCl} = x_3\mathrm{KCl} + x_4\mathrm{MnCl_2} + x_5\mathrm{H_2O} + x_6\mathrm{Cl_2}$$

   Hint: You will need to write down the system of linear equations, then enter in Matlab the augmented matrix. You can use the command *rref* in Matlab to obtain RREF (without doing step by step like in the previous exercise). After that, you need to show your work on how to get x_1, x_2,...,x_6. If there are infinitely many solutions, just pick one (preferably integers).