# Lecture 1 (9/21/2018)

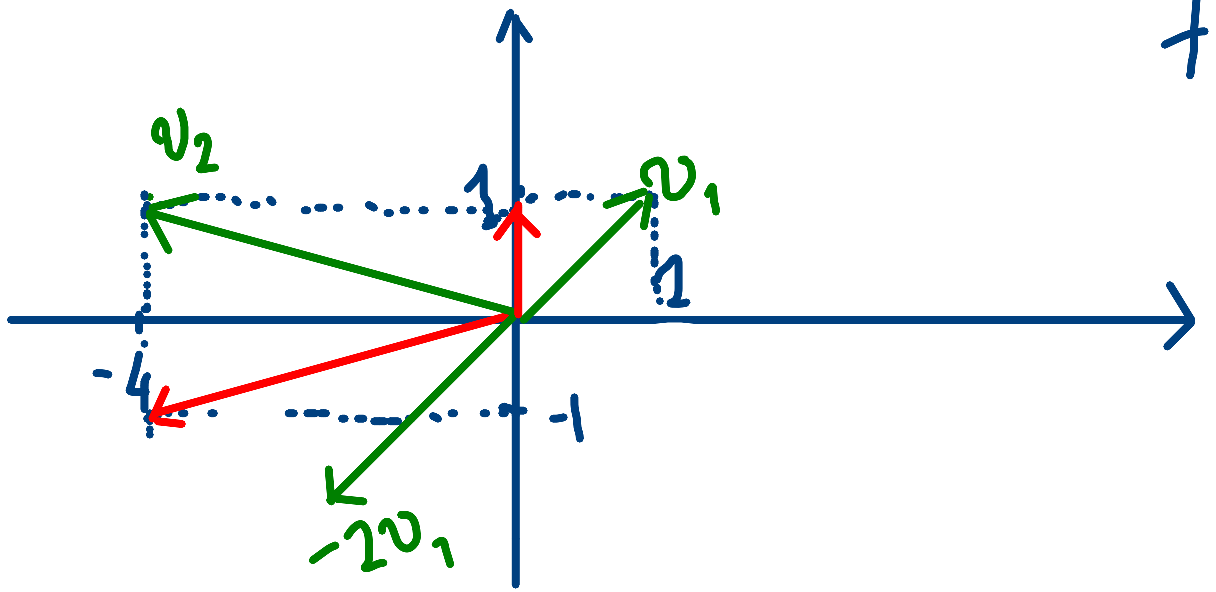
Recurrent themes:

| Calculus | Linear Alg. |
| --- | --- |
| change | structure |
| limit, derivative | linearity, operators/matrices |
| quantification | representation |
| function: rate of change, integral | function ..... linear |
| graph: length, area, volume | matrix: rank, determinant, eigenvalues ... |

linear map / mapping / function / operator / transformation $\dashrightarrow$ same thing

<u>Ex:</u> $f(x,y) = (2x-4y, -x-y)$

$f$ is a map from the plane to itself.




$f(0,1) = (-4,-1)$

$v_1 = (1,1)$ , $v_2 = (-4,1)$ are special vectors because

$f(v_1) = (-2,-2) = -2v_1$

$f(v_2) = (-12,3) = 3v_2$

Any vector in $\mathbb{R}^2$ is a "linear combination" of $v_1$ and $v_2$

$v = \alpha v_1 + \beta v_2$ (for suitable $\alpha$ and $\beta$)

Then $f$ has a simple representation through $v_1$ and $v_2$:

$f(v) = \alpha f(v_1) + \beta f(v_2) = -2\alpha v_1 + 3\beta v_2$

In linear alg., the interest is not so much about computation, but representation and how structures work together.

<u>Ex</u> Balance the following chemical eq.

$$Ba(OH)_2 + H_3PO_4 \longrightarrow H_2O + Ba_3(PO_4)_2$$

In other words, find $x, y, z, t$ (preferably integers) such that

$$x\,Ba(OH)_2 + y\,H_3PO_4 = z\,H_2O + t\,Ba_3(PO_4)_2$$

The number of atoms of each kind should match.

| | left | right |
|---|---|---|
| Ba | $x$ | $3t$ |
| O | $2x+4y$ | $z+8t$ |
| H | $2x+3y$ | $2z$ |
| P | $y$ | $2t$ |

$$\begin{cases} x - 3t = 0 \\ 2x + 4y - z - 8t = 0 \\ 2x + 3y - 2z = 0 \\ y - 2t = 0 \end{cases}$$

This is a system of eqs. of 4 eqs. and 4 unknowns.

How to solve this system?

Let's consider a simpler system: $\begin{cases} x+y=1 \\ 2x-3y=3 \end{cases}$

There are a number of ways to solve:

① Elimination:

(Eq. 1) × 2 − (Eq. 2) : $5y=-1 \Rightarrow y=-1/5$

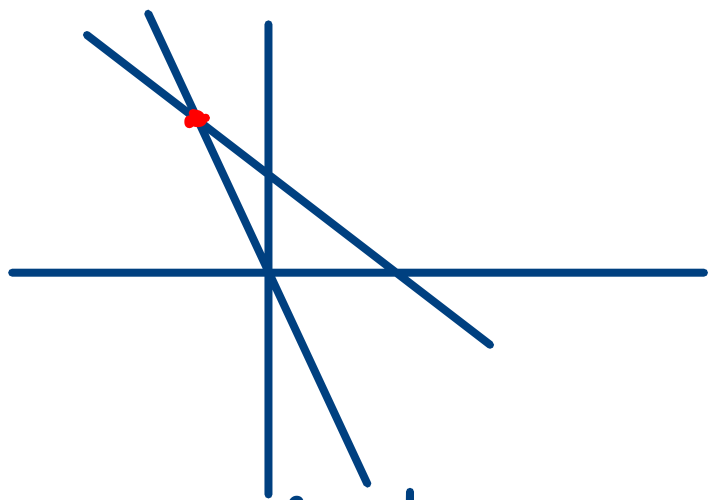
Then $x=1-y=6/5$

② Substitution:

Eq. 1 : $x=1-y$

Eq. 2 : $2(1-y)-3y=3 \rightsquigarrow y=\dots \rightsquigarrow x=\dots$

③ Geometric: each eq. is an eq. of a line on plane. The problem of solving the system is the problem of finding intersection.



The geometric method is not good for bigger systems of eqs. For example, it's not clear why 4 "planes" in 4 dimensions should intersect (or usually intersect at only one point).

The first and second method are more stable under the change of sys. size. They are based on the same idea: reduce the number of unknowns and eqs. (thereby downsize the system). They are algebraic methods.

Formally, we don't care about the unknowns $x$ and $y$, only the coefficients that attach to them. The system is encoded in the following numbers:

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -3 & 3 \end{array}\right]$$

How did we eliminate $x$?

$$R_2 = R_2 - 2R_1 \; : \quad \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -5 & 1 \end{array}\right] \uparrow$$

then we solve from bottom to top

It turns out that this idea is valid/applicable for <u>any</u> system of linear equations. It is known as Gauss elimination method.