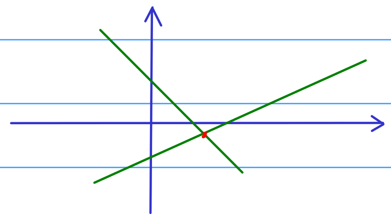
Lecture 2 (9/24/2018)

$$\begin{cases} x + y = 1 \\ 2x - 3y = 3 \end{cases}$$

Two ways to solve — geometric: finding intersection of two lines
— algebra: elimination, substitution



Limitation of geometric method: difficulty to work with bigger systems (more equations, more unknowns)

Representation of the system:

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -3 & 3 \end{array}\right] \quad \text{.... augmented matrix}$$

$\underbrace{\begin{matrix} 2 & -3 \end{matrix}}_{\text{coef. matrix}}$ $\quad \underbrace{3}_{\text{aug. col.}}$

The coef. of x have nothing to do with coef. of y.

Problem: reduce the matrix into simpler form using elementary row operations.

1) $R_i = cR_i$ .... multiply a row by a nonzero number.
2) $R_i = R_i + cR_j$ .... add to a row a multiple of another row.
3) $R_i \leftrightarrow R_j$ .... exchange the order of two rows.

Row echelon form: (looks like staircase)

Ex:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{row echelon form}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{not row echelon form}$$

Def. of row-echelon form matrix
- the first entry of each row is equal to 1. (Called the leading 1)
- For any two successive rows, the leading 1 of the lower row is further to the right than the leading 1 of the row above it.
- the zero rows (if any) are grouped together at the bottom of the matrix.

Ex:

$$\underbrace{\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array}\right]}_{\text{not in row echelon form}} \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 0 & -6 & -10 & -24 \end{array}\right]$$

$$\xrightarrow[R_3 = R_3 + 6R_2]{R_2 = R_2/(-5)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -12 \end{array}\right]$$

$$\xrightarrow{R_3 = R_3/(-4)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$ this is in row echelon form

The above process is called <u>Gauss elimination</u>.

*Issue: row echelon form is not unique, and not suitable to be defined as an intrinsic property of a matrix.

<u>Gauss-Jordan elimination</u>:

- Work from left to right
- Change each column into pivot column, i.e. col. that contains a leading 1, all other entries = 0.

use row operations

- go to the next col. if the current col. can't be made pivot.

Ex:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow[\text{operations}]{\text{row}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

* Reduced row echelon form (RREF)

definition:
- the matrix in row echelon form
- for each column that contains a leading 1, all other entries on that column is equal to 0.

Thm:

RREF of a matrix is unique.

Worksheet handed out