Some review problems for Midterm

1. Solve the following system of linear equations. If a system has infinitely many solutions, write the solutions in parametric vector form.

   (a)
   $$\begin{cases} x_1 + x_2 + x_3 &= 4 \\ -x_1 - x_2 + x_3 &= -2 \\ 2x_1 - x_2 + 2x_3 &= 2 \end{cases}$$

   (b)
   $$\begin{cases} 2x_1 + x_2 - 2x_3 &= 1 \\ x_1 + 2x_2 - 5x_3 &= -2 \\ -x_1 + x_2 - 3x_3 &= -3 \end{cases}$$

   (c)
   $$\begin{cases} 3x_1 + 2x_2 - x_3 - x_4 &= -3 \\ -x_1 + x_3 + 2x_4 &= 1 \\ 2x_1 + 2x_2 + x_4 &= -2 \\ x_1 + 2x_2 + x_3 + 3x_4 &= -1 \end{cases}$$

   (d)
   $$\begin{cases} x_1 + 2x_2 + x_3 &= 2 \\ x_1 + 3x_2 &= 3 \\ x_1 + x_2 + 2x_3 &= 2 \end{cases}$$

2. Determine all values of a such that the matrix
   $$A = \begin{bmatrix} 2 & 0 & 7 & 10 \\ 1 & a & 3 & 3 \\ 1 & -1 & -2 & 1 \end{bmatrix}$$
   has full rank.

3. Is the following matrix invertible? If it is, find the inverse matrix.
   $$\begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

4. Let
   $$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$
   (a) Determine all values of k such that the matrix $A - kI_2$ fails to be invertible.
   (b) Find a nonzero vector v such that $Av = 5v$.

5. Matrix B is said to *commute* with matrix A if $AB = BA$. Find all matrices that commute with $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$. Your answer will contain some arbitrary constant(s).

6. Can the following matrix be the product of elementary matrices? If yes, factor it into elementary matrices. If no, explain why.

   (a)
   $$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$