Quiz 1

10/3/2018

Name: ______________________

Instructions: Show your work. Circle your final answers. The quiz has two pages.

4 pts.

1. Solve the following system of linear equations by Gauss elimination method.

$$\begin{cases} x + 2y + 3z &= 9 \\ 2x - y + z &= 8 \\ 3x - z &= 3 \end{cases}$$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array}\right] \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 0 & -6 & -10 & -24 \end{array}\right]$$

$$\xrightarrow[\substack{R_1 = R_1 - 2R_2 \\ R_3 = R_3 + 6R_2}]{R_2 = R_2/(-5)} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -12 \end{array}\right]$$

$$\xrightarrow[\substack{R_1 = R_1 - R_3 \\ R_2 = R_2 - R_3}]{R_3 = R_3/(-4)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \text{(RREF)}$$

Therefore, $x = 2,\ y = -1,\ z = 3.$

4 pts. 2. Determine the rank of matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & -2 & 1 & 0 \\ 1 & -5 & 0 & -4 \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 - R_1]{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -1 & -4 \\ 0 & -6 & -2 & -8 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in row echelon form (REF). The number of nonzero rows is 2.

$\text{rank}(A) = 2.$

2 pts. 3. Can a system of linear equations with 2 equations and 3 unknowns have a unique solution? If yes, give an example of such a system. If no, explain why.

In order for a system to have a unique solution, its RREF must satisfy two following conditions:

- There are no rows of the form $[0\ 0\ 0 \mid a]$ ← nonzero
- All columns (on the right of the bar) must be pivot columns.

$$\left[\begin{array}{ccc|c} * & * & * & * \\ * & * & * & * \end{array}\right]$$

The second condition cannot be satisfied in our case, because there is always at least one nonpivot column.

No