Quiz 2

10/10/2018

Name: ______________________________

Instructions: Show your work. Circle your final answers. The quiz has two pages.

2pts 1. Suppose A_1, A_2, A_3, A_4 are invertible matrices of the same size. Is matrix

$$A = 2A_1A_2^{-1}A_3^{-1}A_4$$

invertible? If so, write A^{-1} in terms of A_1, A_2, A_3, A_4. If not, give a counterexample.

Yes, A is invertible.

$$A^{-1} = \frac{1}{2} A_4^{-1} A_3 A_2 A_1^{-1}$$

4pts 2. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[A|I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 = R_3 - R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 = R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{R_2 = R_2 + R_3 \\ R_3 = -R_3}]{R_1 = R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

2 pts 3. Let A and D be two 3-by-3 matrices. Suppose A can be transformed into D by the following elementary row operations:

$$A \xrightarrow{R_1=R_1+2R_2} B \xrightarrow{R_3=2R_3} C \xrightarrow{R_2\leftrightarrow R_3} D$$

(a) What is the reverse chain that takes D back to A?

$$D \xrightarrow{R_2 \leftrightarrow R_3} C \xrightarrow{R_3=\frac{1}{2}R_3} B \xrightarrow{R_1=R_1-2R_2} A$$

2 pts (b) If $D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 1 & -1 & 2 \end{bmatrix}$, is A invertible? Explain why or why not.

No. A is row equivalent to D.

D has a zero row. Then the RREF of D also has a zero, which can't be equal to the identity matrix.

D is not invertible $\Rightarrow$ A is not either.