Quiz 3

10/17/2018

Name: _____

Instructions: Show your work. Circle your final answers. The quiz has two pages.

1. Let A, B, C be 2-by-2 matrices with determinant $|A| = 1$, $|B| = 2$, $|C| = 3$.

1 pt (a) What is the determinant of $3B$?

$|3B| = 3^2 |B| = 9 \times 2 = 18$

2 pts (b) What is the determinant of $2A^2B^{-1}C$?

$|2A^2B^{-1}C| = 2^2 |A|^2 |B|^{-1} |C| = 4 \times 1^2 \times \frac{1}{2} \times 3 = 6$

4 pts 2. Determine all values of a such that the following matrix is invertible

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & a & 2 \\ -1 & -1 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ 0 & a \\ -1 & -1 \end{matrix}$$

$\det = (a - 4 + 0) - (-3a - 2 + 0) = a + 3a - 4 + 2 = 4a - 2$

The matrix is invertible if and only if $\det \neq 0$.

That is, $a \neq \frac{1}{2}$.

3 pts 3. Let A and B be 2-by-2 matrices. Is the equality

$$\det(A+B) = \det A + \det B$$

always true? If yes, explain why. If no, give a counterexample.

No. For example, $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = 1$, $|B| = 1$

$A + B = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$

$|A+B| = 4 \neq |A| + |B|$

(there are a lot of other counterexamples.)