

Quiz 4

10/31/2018

Name: _____

Instructions: Show your work. Circle your final answers. The quiz has two pages.

The matrix $A = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 2 & 2 & 10 & -6 \\ 1 & 0 & 3 & -2 \end{bmatrix}$ has reduced row echelon form $B = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

1. What is the rank of A ?

(1 pt) $\text{rank}(A) = 2$ (# non zero rows of $B = \#$ pivot cols. of B)

2. Determine a basis of the column space of A . What is its dimension?

(2 pt) $\left\{ \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$ $\dim C(A) = 2$

3. Determine a basis of the row space of A . What is its dimension?

(2 pt) $\left\{ (1, 0, 3, -2), (0, 1, 2, -1) \right\}$ $\dim R(A) = 2$

4. Supplement more vectors to the basis of the row space which you obtain in Part 3 to get a basis for \mathbb{R}^4 .

(2pt) add these vectors: $(0, 0, 1, 0)$,
 $(0, 0, 0, 1)$.

5. Determine a basis for the null space of A . What is its dimension? (In other words, what is the nullity of A ?)

(3pt)

$$B = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\uparrow \quad \uparrow$
 nonpivot

$$x_4 = t, \quad x_3 = s$$

From the second row of B : $x_2 + 2x_3 - x_4 = 0$
 $\Rightarrow x_2 = -2s + t$

From the first row: $x_1 + 3x_3 - 2x_4 = 0$
 $\Rightarrow x_1 = -3s + 2t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3s + 2t \\ -2s + t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Basis: $\left\{ \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$\dim N(A) = 2.$$