

## Quiz 5

11/07/2018

Name: \_\_\_\_\_

**Instructions:** Show your work. Circle your final answers. The quiz has two pages.

1. Consider a linear map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,

$$f(x, y, z) = (x + z, y + 2z, -2x + y).$$

- 2pt (a) Write the matrix representing  $f$ .

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ -2 & 1 & 0 \end{bmatrix}$$

- 3pt (b) Given that the reduced row echelon form of the matrix in Part (a) is

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Determine a basis of  $\ker(f)$  and the dimension. Is  $f$  injective?

nonpivot column

$$\begin{cases} x_3 = t \\ x_2 = -2t \\ x_1 = -t \end{cases} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t \\ -2t \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$\ker(f)$  is 1-dimensional, with basis  $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right\}$

$f$  is not injective.

3pt (c) Determine a basis of  $\text{range}(f)$  and the dimension. Is  $f$  surjective?

The first and second column of  $A$  form a basis for  $\text{range}(f)$ :

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} \quad \text{range}(f) \text{ is 2-dimensional.}$$

$f$  is not surjective.

2pt 2. Let

$$E = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -2 \end{bmatrix} \quad 2 \times 3 \text{ matrix}$$

What is the linear map matrix  $E$  represents? (Write the domain, target set, and an explicit formula of the map.)

$$f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \underbrace{\begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -2 \end{bmatrix}}_E \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x + y + 2z \\ 3x - 2z \end{bmatrix}$$