Quiz 6

11/14/2018

Name: ____________________________

Instructions: Show your work. Circle your final answers. The quiz has two pages.

1. Consider a basis $S = \{v_1, v_2, v_3\}$ of $\mathbb{R}^3$ where

2 pt

$$\begin{aligned} v_1 &= (2, 6, 5) \\ v_2 &= (5, 3, -2) \\ v_3 &= (7, 4, -3) \end{aligned}$$

Set up a formula to compute the coordinate of vector $b = (2, 3, 1)$ in basis S. (You don't need to compute matrix inverse or matrix multiplication. Only set up a correct formula for $[b]_S$.)

$$[b]_S = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix}^{-1} [b]_{S_0} = \begin{bmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

2. Consider the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 6 & 3 \end{bmatrix}$$

2pt (a) Compute $\det(A - \lambda I_2)$.

$$\det(A - \lambda I_2) = \begin{vmatrix} 4-\lambda & 1 \\ 6 & 3-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda) - 6 = \lambda^2 - 7\lambda + 6$$

(b) Compute the eigenvalues of A. Label them by λ_1 and λ_2 such that $\lambda_1 < \lambda_2$.

2pt

$\lambda^2 - 7\lambda + 6 = (\lambda - 1)(\lambda - 6)$ ...... has two roots

$\lambda_1 = 1 \,, \quad \lambda_2 = 6$

(c) Determine $E(\lambda_1)$, the eigenspace corresponding to λ_1. Determine a basis and the dimension.

2pt

null space of $A - I_2 = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$

$$\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \xrightarrow[R_1 = R_1/3]{R_2 = R_2 - 2R_1} \begin{bmatrix} 1 & 1/3 \\ 0 & 0 \end{bmatrix}$$

non pivot column

$x_2 = t$

$x_1 = -t/3$

$$E(\lambda_1) = \left\{ \begin{bmatrix} -t/3 \\ t \end{bmatrix} : t \in \mathbb{R} \right\} = \text{span}\left\{ \begin{bmatrix} -1/3 \\ 1 \end{bmatrix} \right\}$$

basis

$\dim E(\lambda_1) = 1$

2pt (d) Determine $E(\lambda_2)$, the eigenspace corresponding to λ_2. Determine a basis and the dimension.

null space of $A - 6I_2 = \begin{bmatrix} -2 & 1 \\ 6 & -3 \end{bmatrix}$

$$\begin{bmatrix} -2 & 1 \\ 6 & -3 \end{bmatrix} \xrightarrow[R_1 = R_1/(-2)]{R_2 = R_2 + 3R_1} \begin{bmatrix} 1 & -1/2 \\ 0 & 0 \end{bmatrix}$$

non pivot column

$x_2 = t$

$x_1 = t/2$

$$E(\lambda_2) = \left\{ \begin{bmatrix} t/2 \\ t \end{bmatrix} : t \in \mathbb{R} \right\} = \text{span}\left\{ \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \right\}$$

basis

$\dim E(\lambda_2) = 1$