Quiz 7

11/21/2018

Name: ______________________________

Instructions: Show your work. Circle your final answers. The quiz has two pages.

1. Consider a basis $S = \{v_1, v_2, v_3\}$ of $\mathbb{R}^3$ where

$$\begin{aligned} v_1 &= (2, 6, 5) \\ v_2 &= (5, 3, -2) \\ v_3 &= (7, 4, -3) \end{aligned}$$

Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map satisfying

$$\begin{aligned} f(v_1) &= v_1 + v_2 \\ f(v_2) &= v_2 + v_3 \\ f(v_3) &= v_1 + v_3 \end{aligned}$$

2pt (a) Write the matrix representing f in basis S.

$$[f]_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

2pt (b) Set up a correct formula for the matrix A representing f in the standard basis. You don't need to compute matrix inverse or matrix multiplication. Your answer should not contain any letters.

$$P = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix}$$

$$\begin{tikzcd} x \arrow[r, "A"] \arrow[d, "P^{-1}"'] & y \\ {[x]_S} \arrow[r, "{[f]_S}"] & {[y]_S} \arrow[u, "P"'] \end{tikzcd}$$

$$A = P[f]P^{-1} = \begin{bmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{bmatrix}^{-1}$$

2. Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

2pt (a) Determine all eigenvalue(s) of A together with their multiplicity. *Hint: the matrix is already in upper triangular form.*

$\lambda = 2$ (the only entry only the diagonal)

multiplicity $= 3$

2pt (b) Determine the eigenspace(s) of A. What is its dimension?

$$A - \lambda I = A - 2I = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

nonpivot cols

$x_1 = s$

$x_3 = t$

$x_2 = 0$

$$E(\lambda) = \left\{ \begin{bmatrix} s \\ t \\ 0 \end{bmatrix} : s, t \in \mathbb{R} \right\} = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$\dim E(\lambda) = 2$

2pt (c) Is A diagonalizable? If yes, diagonalize it. If no, explain why.

No. The dimension of $E(\lambda)$ is less than the multiplicity of λ.

(there are only 2 lin. ind. eigenvectors, not sufficient to form a basis for $\mathbb{R}^3$).