

Homework 2

Due 10/11/2019

In the following problems, make sure to write your arguments coherently in full sentences.

1. Let V be a vector space over a field $F = \mathbb{Q}, \mathbb{R}, \mathbb{C}$. Suppose $v_1, v_2 \in V$ are linearly independent of each other. Show that vectors $v_1 + 2v_2$ and $2v_1 + 3v_2$ are linearly independent.
2. Show that the functions $y_1 = \sin x$, $y_2 = \cos x$ and $y_3 = \sin(2x)$ are linearly independent over \mathbb{R} .
3. Consider the set

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, a + d = 0 \right\}.$$

- (a) Show that V is a vector space over \mathbb{R} .
- (b) Find a basis of V .

Do the following problem for 6 bonus points.

4. Are the functions $y_1 = \sin x$, $y_2 = \cos x$ and $y_3 = \sin(x + 1)$ linearly independent over \mathbb{R} ? Verify your answer.