

Homework 5

Due 11/08/2019

In the following problems, make sure to write your arguments coherently in full sentences. If possible, start a sentence with words rather than a formula. Avoid using ambiguous symbols such as \rightarrow , $?$, \dots , \therefore . Instead, use words to transition your ideas, for example “This leads to”, “Therefore”, “We want to show”, etc.

Consider the following subspaces of \mathbb{R}^3 :

$$\begin{aligned}U &= \{(x_1, x_2, x_3) : x_1 = x_2 + x_3\}, \\V &= \{(x_1, x_2, x_3) : x_1 = x_2\}, \\W &= \{(x_1, x_2, x_3) : x_1 = x_2 = x_3\}.\end{aligned}$$

1. Find a basis of the intersection $U \cap V$. What is the dimension? Convention: the basis of the $\{0\}$ vector space is \emptyset (the empty set) and its dimension is 0.
2. Find a basis of $U \cap W$. What is the dimension?
3. Show that $U + W = \mathbb{R}^3$.

Hint: draw a picture of U and W . Show that $U + W$ contains a basis of \mathbb{R}^3 .

Do the following problem for 6 bonus points.

4. Show that $V + W = V$.