Homework 6

Due 11/15/2019

In the following problems, make sure to write your arguments coherently in full sentences. If possible, start a sentence with words rather than a formula. Avoid using ambiguous symbols such as \rightarrow , ?, ..., \therefore Instead, use words to transition your ideas, for example "This leads to", "Therefore", "We want to show", etc.

- 1. Let V be a vector space over a field of numbers F (which could be \mathbb{Q} , \mathbb{R} or \mathbb{C}). Let U be a subspace of V. Show that U + U = U. Under what condition of U is this sum a direct sum?
- 2. Let $U = \{(x, y, y, x) : x, y \in \mathbb{R}\}$. This is a subspace of \mathbb{R}^4 . Find a subspace V of \mathbb{R}^4 such that $U \oplus V = \mathbb{R}^4$.
- 3. Consider two vector spaces

$$U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, a + d = b + c = 0 \right\},$$
$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, a = d = 0 \right\}.$$

(a) Find a basis of U + V.

Hint: You can use Matlab to compute RREF of a matrix. (Make sure to write the Matlab code on your homework. See lecture note on 11/08 for an example.) If you choose not to use Matlab, make sure that you write all row reduction steps.

(b) Is U + V a direct sum?

Do the following problem for 6 bonus points.

4. Let $V = \{z \in \mathbb{C} : z(1+i) + 2\overline{z} = 0\}$. Is V a vector space over \mathbb{C} ?