## Homework 6

Due 11/15/2019

In the following problems, make sure to write your arguments coherently in full sentences. If possible, start a sentence with words rather than a formula. Avoid using ambiguous symbols such as $\rightarrow, ?, \ldots, \therefore$ Instead, use words to transition your ideas, for example "This leads to","Therefore", "We want to show", etc.

1. Let $V$ be a vector space over a field of numbers $F$ (which could be $\mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$ ). Let $U$ be a subspace of $V$. Show that $U+U=U$. Under what condition of $U$ is this sum a direct sum?
2. Let $U=\{(x, y, y, x): x, y \in \mathbb{R}\}$. This is a subspace of $\mathbb{R}^{4}$. Find a subspace $V$ of $\mathbb{R}^{4}$ such that $U \oplus V=\mathbb{R}^{4}$.
3. Consider two vector spaces

$$
\begin{aligned}
U & =\left\{\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right]: a, b, c, d \in \mathbb{R}, a+d=b+c=0\right\} \\
V & =\left\{\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right]: a, b, c, d \in \mathbb{R}, a=d=0\right\} .
\end{aligned}
$$

(a) Find a basis of $U+V$.

Hint: You can use Matlab to compute RREF of a matrix. (Make sure to write the Matlab code on your homework. See lecture note on $11 / 08$ for an example.) If you choose not to use Matlab, make sure that you write all row reduction steps.
(b) Is $U+V$ a direct sum?

Do the following problem for 6 bonus points.
4. Let $V=\{z \in \mathbb{C}: z(1+i)+2 \bar{z}=0\}$. Is $V$ a vector space over $\mathbb{C}$ ?

