Homework 8

Due 12/04/2019

In the following problems, make sure to write your arguments coherently in full sentences. If possible, start a sentence with words rather than a formula. Avoid using ambiguous symbols such as $?, \ldots, \therefore$. You can use the arrows to indicate row operations. In other circumstances, use words to transition your ideas, for example "This leads to", "Therefore", "We want to show", etc.

1. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map given by f(v) = Av where

$$A = \begin{bmatrix} 3 & -2 & -2 \\ -1 & 4 & 2 \\ 2 & -4 & -2 \end{bmatrix}$$

Is f diagonalizable? If it is, express $V = \mathbb{R}^3$ as a direct sum of one-dimensional invariant subspaces under f; then find a basis of V in which f is represented by a diagonal matrix.

2. Let $f: M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be a linear map given by

$$f\left(\begin{bmatrix}a&b\\c&d\end{bmatrix}\right) = \begin{bmatrix}a&2a+b\\2c&c+2d\end{bmatrix}.$$

Is f diagonalizable? If it is, express $V = M_{2\times 2}(\mathbb{R})$ as a direct sum of one-dimensional invariant subspaces under f; then find a basis of V in which f is represented by a diagonal matrix.

3. Let V be the vector space of all smooth (infinitely differentiable) functions from \mathbb{R} to \mathbb{R} . Let $F: V \to V$ be a linear map defined by F(u) = u'. Find all eigenvectors and eigenvalues of F.

Do the following problem for 6 bonus points.

4. Let $f: \mathbb{C}^2 \to \mathbb{C}^2$ be a linear map given by f(v) = Av where

$$A = \begin{bmatrix} 1-i & 2-i \\ 0 & 2+i \end{bmatrix}.$$

Is f diagonalizable? If it is, express $V = \mathbb{C}^2$ as a direct sum of one-dimensional invariant subspaces under f; then find a basis of V in which f is represented by a diagonal matrix.