Some review problems for Midterm

In the following problems, verify your answer with valid arguments. Make sure to write in full sentences.

1. Check if the set V given below is a vector space.
   (a) V is the set of all 2×2 matrices with real coefficients that have vanishing determinant.
   (b) V is the set of all functions from $\mathbb{R}$ to $\mathbb{R}$ that vanish at 1 and 2.

2. Let $F : \mathbb{R}^2 \to P_1$ and $G : P_1 \to M_{2\times 2}(\mathbb{R})$ be given as

$$F(a, b) = 2ax - b, \qquad G(u) = \begin{bmatrix} u(1) & u(0) \\ u(0) & u(-1) \end{bmatrix}.$$

   Here P_1 denotes the set of all polynomials of degree ≤ 1 with real coefficients.
   (a) Show that G is a linear map.
   (b) Find a matrix representation of F, G and the composite map $G \circ F$.

3. Let $F : P_2 \to P_2$ be defined by $F(u) = xu'$. Here P_2 denotes the set of all polynomials of degree ≤ 2 with real coefficients.
   (a) Show that F is a linear map.
   (b) Find a matrix representation of F.
   (c) Find a basis of null(F). What is the nullity of F?
   (d) Find a basis of range(F). What is the rank of F?
   (e) Is F monomorphic, epimorphic, isomorphic, or none of them? Verify your answer.