Worksheet

10/14/2019

Consider a linear map $f : \mathbb{R}^2 \to M_{2\times 2}(\mathbb{R})$ given by

$$f(a,b) = \begin{bmatrix} a & a+b \\ a-b & b \end{bmatrix}$$

Write a matrix that represents f by following the procedure:

- Find a basis for $\mathbb{R}^2$.
- Find a basis for $M_{2\times 2}(\mathbb{R})$.
- Compute each column of $[f]_{B_2,B_1}$.

$V = \mathbb{R}^2$ has a basis $B_1 = \{e_1, e_2\}$ where $e_1 = (1,0)$, $e_2 = (0,1)$.

$W = M_{2\times 2}(\mathbb{R})$ has a basis $B_2 = \{E_1, E_2, E_3, E_4\}$ where

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The matrix representation of f in basis B_1, B_2 is

$$[f]_{B_2,B_1} = \begin{bmatrix} | & | \\ [f(e_1)]_{B_2} & [f(e_2)]_{B_2} \\ | & | \end{bmatrix}$$

We have $f(e_1) = f(1,0) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = E_1 + E_2 + E_3$.

Then $[f(e_1)]_{B_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.

We have $f(e_2) = f(0,1) = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = E_2 - E_3 + E_4$.

Then $[f(e_2)]_{B_2} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$.

Conclusion: $[f]_{B_2,B_1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$