

Worksheet
10/30/2019

1. Let V be the set of all functions from \mathbb{R} to \mathbb{R} that vanish at 1 and 2. Check if V a vector space over \mathbb{R} .

2. Let $F : \mathbb{R}^2 \rightarrow P_1$ and $G : P_1 \rightarrow M_{2 \times 2}(\mathbb{R})$ be given as

$$F(a, b) = 2ax - b, \quad G(u) = \begin{bmatrix} u(1) & u(0) \\ u(0) & u(-1) \end{bmatrix}.$$

Here P_1 denotes the set of all polynomials of degree ≤ 1 with real coefficients.

(a) Find a matrix representation of F .

(b) Find a matrix representation of G .

(c) Find a matrix representation of $G \circ F$.

3. Let $F : P_2 \rightarrow P_2$ be defined by $F(u) = xu'$. Here P_2 denotes the set of all polynomials of degree ≤ 2 with real coefficients.

(a) Show that F is a linear map.

(b) Find a matrix representation of F .

(c) Find a basis of $\text{null}(F)$. What is the nullity of F ?

(d) Find a basis of $\text{range}(F)$. What is the rank of F ?