Some review problems for Final

1. Find the polynomial the fits the following points by Lagrange and Newton methods

   (a) $(-1,1)$, $(0,-1)$, $(1,1)$, $(2,0)$.

   (b) $(-1,0)$, $(0,-1)$, $(1,0)$, $(0,1)$.

2. Let $f(x) = \frac{1}{2+3x}$. For evenly spaced sample points $1 = x_1 < x_2 < \ldots < x_n = 2$, let P_n be the corresponding interpolation polynomial. Find n such that the integral $\int_1^2 P_n(x)dx$ approximates the integral $\int_1^2 f(x)dx$ with error not exceeding 10^{-4}.

3. We want to find an approximate value of the integral $I = \int_2^4 \frac{1}{x^3+1}dx$. Let n be the number of equal subintervals of the interval $[2,4]$.

   (a) For $n = 5$, use right-point rule to approximate I.

   (b) For $n = 5$, use midpoint rule to approximate I.

   (c) For $n = 5$, use trapezoid rule to approximate I.

   (d) For $n = 5$, use Simpson rule to approximate I.

   (e) How big should n be such that the approximate value of I by midpoint rule is under 10^{-4} ?

   (f) The same question as Part (e) but for Simpson rule.