Homework 2

Due 10/11/2019

1. Let $f(x) = xe^{-x^2}$.

   (a) Find the degree $2n+1$ Taylor polynomial for $f(x)$, about the point $x_0 = 0$.
   Hint: use the identity with $t = -x^2$

   $$e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots + \frac{t^m}{m!} + R_m(t).$$

   (b) Bound the error in degree $2n+1$ approximation for $|x| \leq 2$.
   Hint: use Lagrange theorem to bound the term $R_n(t)$ *where* $t = -x^2$.

   (c) Find n so as to have $2n+1$ Taylor approximation with error of at most 10^{-9} on $[-2, 2]$.

2. Convert the number $(101.011)_2$ from binary system to decimal system. *(Make sure to show all your calculations, not just the final result.)*

3. Convert the number 3.7 from decimal system to binary system. *(Make sure to show all your calculations, not just the final result.)*

In the following problems, use the floating-point format described in Worksheet 10/4/2019 (handed in class, also posted on Canvas and the course website).

4. Do the following operations. Write your results in both floating-point format and decimal format. *Make sure to show all your calculations, not just the final result.*

   (a) $(1.001)_2 \times 2^2 + (1.101)_2 \times 2^4$

   (b) $(1.001)_2 \times 2^1 - (1.101)_2 \times 2^3$

   (c) $(1.001)_2 \times 2^7 + (1.101)_2 \times 2^7$

   (d) $(1.001)_2 \times 2^6 + (1.100)_2 \times 2^{-2}$
   What do you notice when adding these two numbers of quite different size?

5. What number does the bit sequence 1 0 0 1 1 0 1 1 represent?

6. What is the smallest number greater than 1 that can be represented by floating-point format? Call this number b. The difference $\epsilon = b - 1$ is called the *machine epsilon* of this number format. Find ϵ.