Homework 4

Due 10/25/2019

We know that a general quintic equation cannot be solved by radicals (Abel–Ruffini 1820s). However, we can find approximate roots within any prescribed error by bisection or Newton method. In Problems 1 through 6, let $f(x) = x^5 - 3x^2 + 1$.

1. Use Intermediate Value Theorem to show that f has a root in each of the intervals $(-1, 0)$, $(0, 1)$, $(1, 2)$. Label these roots by r_1, r_2, r_3 respectively.

2. Use Matlab to plot the graph of f on the interval $(-1, 2)$.

3. With the help of your pocket calculator, use bisection method to compute approximately root r_1, with the initial interval $(a_0, b_0) = (-1, 0)$, after 4 iterations.

4. Regarding to the previous problem, how many iterations are needed in order to compute r_1 with error under 10^{-6} ? The interactive applet available at https://www.geogebra.org/m/XndvAujc can help you visualize the bisection method.

5. Write a Matlab program to compute approximately r_1, r_2, r_3 by Newton's method. This program should contain a 'while' loop which stops when $|x_{n+1} - x_n| \leq 10^{-6}$. The initial point is of your choice. (You may want to write 3 separate programs, one for each root.)

6. With each initial point $x_0 = 0.16$, 0.17, 0.18, 0.19, which root does your program give? The interactive applet available at https://www.geogebra.org/m/DGFGBJyU can help you visualize Newton's method.

7. Let $f(x) = \sqrt[3]{x}$. We know that $x = 0$ is the only root of f. Nevertheless, we want to test if Newton's method is able to give us this root.

   (a) Plot the function f on the interval $[-5, 5]$.

   (b) Write the iterative formula of the Newton method.

   (c) Express x_n as a function of n and x_0.

   (d) For what x_0 does x_n converge? Is Newton's method a good method to find root of f ?