

## Homework 5

Due 11/08/2019

1. Let  $x_n$  be a sequence defined recursively by  $x_{n+1} = \frac{1}{4}(x_n^3 - 3x_n + 6)$ .
  - (a) Suppose  $x_n$  has a limit  $a \in \mathbb{R}$ . Find all possible values of  $a$ .
  - (b) What is the function  $g$  such that  $x_{n+1} = g(x_n)$ ? Graph this function (using Matlab) on the interval  $(-4, 3)$ . Find all the fixed points of  $g$ .
  - (c) Let  $x_0 = 1.5$ . Draw roughly (by hand) a cobweb diagram of the sequence  $x_n$ . The website <https://www.geogebra.org/m/QJ79IWCL> can be a helpful tool.
  - (d) Which of the fixed points are stable? Which are unstable? Recall:  *$r$  is a stable fixed point if the sequence  $x_n$  converges to  $r$  as long as  $x_0$  is chosen close to  $r$  (but not equal to  $r$ ).*
  - (e) Given that the sequence  $x_n$  converges to 1 as  $n \rightarrow \infty$ , find the order of convergence of  $x_n$ .
2. In this problem, we want to find a polynomial curve passing through four points  $(1, 0)$ ,  $(2, 2)$ ,  $(3, 0)$ ,  $(4, 1)$ .
  - (a) Find a polynomial  $P$  whose graph passes through the given points. Make sure to simplify  $P$ .
  - (b) Use Matlab to plot the graph of  $P$  on the interval  $(0, 5)$ .
  - (c) What is the position on this curve when  $x = 1.5$ ? What is the slope at this point? In other words, find  $P(1.5)$  and  $P'(1.5)$ .