Homework 6

Due 11/18/2019

1. In this problem, you can use the Matlab program posted on course website and Canvas (also mentioned in the lecture notes on 11/08) that computes the interpolation polynomial. We want to see how well a given function can be approximated by the interpolation polynomials. Let f be some function. On the interval $[-5,5]$, take N *equally spaced* points $-5 = x_1 < x_2 < \ldots < x_N = 5$. Take N points $(x_1, y_1), \ldots (x_N, y_N)$ on the graph of f.

   (a) For $f(x) = \sin x$, plot the graph of the interpolation P on the interval $[-5,5]$ in the case $N = 3$, $N = 6$, $N = 11$, $N = 21$. What do you notice? Does the interpolation polynomial approximate well the function f on the interval $[-5,5]$ when N gets larger?
   Hint: You can plot f and all of P's on the same graph by using the command 'hold on'.

   (b) The same questions as in Part (a) but for $f(x) = \frac{1}{1+10x^2}$.

2. Use Newton formula to find a polynomial of degree ≤ 3 that fits the following points $(2,1)$, $(1,0)$, $(3,-1)$, $(0,2)$. Convert the polynomial into the standard form $P(x) = ax^3+bx^2+cx+d$. *Hint:* if you don't want to simplify the polynomial by hand, you can use the command **simplify** of Matlab.

3. Reorder the points in Problem 1 as follows: $(3,-1)$, $(1,0)$, $(0,2)$, $(2,1)$. Find the Newton formula corresponding to these data points (in this order). Do you get the same polynomial as in Problem 1? Explain your observation.

4. You are recommended to do Matlab Practice 3 (posted on course website and Canvas) before starting this problem.
   Write a function in Matlab that does the following:

   - Input:
     - a function f,
     - an array x, i.e. a vector $x = (x_1, x_2, \ldots, x_n)$.
   - Output: the divided difference $f[x_1, x_2, \ldots, x_n]$.

   Test your function with $f(t) = \frac{1}{1+t^2}$ and $x = (1,2,3,4)$.

5. Use the function you wrote in Problem 3 to write a Matlab program (in a script file) that compute the polynomial of degree ≤ 4 that fits the data points $(2,1)$, $(1,0)$, $(3,-1)$, $(0,2)$, $(4,0)$.