Homework 8

Due 12/04/2019

Denote $I = \int_0^2 \frac{1}{4+x^2}\,dx$.

1. Find the exact value of I (for example, by finding an antiderivative of the integrand).

2. For a generic positive integer n, we take $n+1$ equally spaced sample points indexed by $x_0, x_1, \ldots, x_n$ on the interval $[0,2]$. Denote by L_n, R_n, M_n, T_n the Riemann sums corresponding to left-point, right-point, midpoint and trapezoid rule. Use sigma notation to write a formula for each L_n, R_n, M_n, T_n.

3. Which of these 4 methods (rules) gives the best approximation of I when $n = 4$?

4. Write Matlab codes to compute L_n, R_n, M_n, T_n when $n = 8$, 16, 32, 64.

5. Denote by $e_n^{(L)} = |L_n - I|$ the error term from left-point rule. We use similar notations for $e_n^{(R)}$, $e_n^{(M)}$, $e_n^{(T)}$. It is known that

$$e_n^{(L)},\, e_n^{(R)} \leq \frac{K(b-a)^2}{2n}, \quad e_n^{(M)} \leq \frac{\tilde{K}(b-a)^3}{24n^2}, \quad e_n^{(T)} \leq \frac{\tilde{K}(b-a)^3}{12n^2}$$

   where $K = \max_{[a,b]} |f'(x)|$ and $\tilde{K} = \max_{[a,b]} |f''(x)|$. Find n such that the left-point rule gives an error not exceeding $\epsilon = 0.0001$. The same question for the right-point, midpoint, trapezoid rule.
   Hint: you don't need to find the exact values of K and $\tilde{K}$. An upper bound for each would be sufficient for our need.