

# Optional Assignment

Due 12/06/2019

In this problem, we want to find the empirical rate of convergence of the left-point and trapezoid rule. Let  $[a, b] = [1, 4]$ . For  $f(x) = 1 + 2x$ , put  $I = \int_a^b f(x)dx$  and  $e_n^{(L)} = |L_n - I|$ . Let  $h$  be the mesh size.

- (a) For each  $h = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ , find the corresponding error term  $e_n^{(L)}$ .
- (b) Plot the log-log diagram. That is, the plot of 5 points with coordinate  $(\ln h, \ln e_n^{(L)})$ .
- (c) Use the command *polyfit* in Matlab to find the best line that fits these 5 points on the log-log diagram.
- (d) What is the empirical rate of convergence of the sequence  $L_n$  to  $I$ ? That is, the number  $\alpha$  such that  $e_n^{(L)} \sim h^\alpha$ . Compare it to the theoretical rate of convergence.
- (e) The same questions in Part (a)–(d) but with  $f(x) = x^2 - 4x + 5$ .