Worksheet

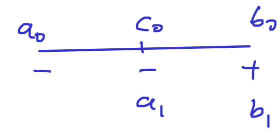
10/14/2019

Name: ______________________________

1. Find a real root of $x^3 - 2x - 2 = 0$ by bisection method: start with $[a_0, b_0] = [0, 2]$. Find $[a_4, b_4]$. What is an estimate of the error if one takes $x_0 = (a_4 + b_4)/2$?

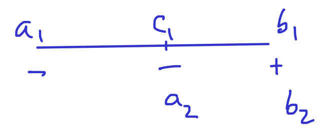
$a_0 = 0$, $b_0 = 2$, $c_0 = \frac{a_0 + b_0}{2} = 1$

$f(a_0) < 0$, $f(b_0) > 0$, $f(c_0) < 0$



Then $a_1 = c_0 = 1$, $b_1 = b_0 = 2$, $c_1 = \frac{a_1 + b_1}{2} = 1.5$

$f(a_1) = f(c_0) < 0$, $f(b_1) > 0$, $f(c_1) < 0$



Then $a_2 = c_1 = 1.5$, $b_2 = b_1 = 2$, $c_2 = \frac{a_2 + b_2}{2} = 1.75$

(Continue this procedure ....)

2. Find a root with error tolerance $\epsilon = 10^{-3}$.

$n \geq \log_2\left(\frac{b_0 - a_0}{\varepsilon}\right) - 1 = \log_2\left(\frac{2-0}{10^{-3}}\right) - 1 \approx 9.96$

Thus, by taking $x_0 = \frac{a_{10} + b_{10}}{2}$, one obtains an approximate root with precision ε.

To find a_{10} and b_{10}, one continues the procedure in Problem 1 six more times.