Worksheet

10/18/2019

Name: ______________________________

1. Let us compute approximately $\sqrt[3]{3}$ by Newton's method by following the procedure:

   - Find a function f such that $x_* = \sqrt[3]{3}$ is a root.
   - Write the iteration formula of Newton's method.
   - Pick a point x_0 as the initial iteration. The closer x_0 is to x_* the better.
   - Do four iterations. What do you get for x_4?

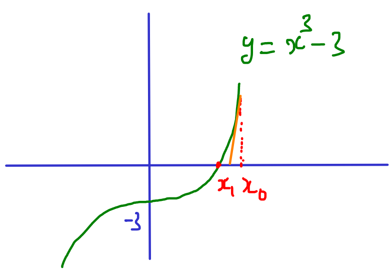
One can choose $f(x) = x^3 - 3$.

$f'(x) = 3x$.

The iteration formula for Newton's method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 3}{3x_n^2} = \frac{2x_n}{3} + \frac{1}{x_n^2}$$

One can take $x_0 = 2$.



$$x_1 = \frac{2 \times 2}{3} + \frac{1}{2^2} \approx \cdots$$

$$x_2 = \cdots$$

2. Find approximately the intersection point of the graph of $u(x) = e^{-x}$ and the graph of $v(x) = 2xe^x$.

Set $u(x) = v(x)$ and solve for x.

$$e^{-x} = 2x e^{x}$$

This is equivalent to

$$1 = 2x e^{2x}$$

Put $t = 2x$. We solve for t from $t e^{t} = 1$.

Put $f(t) = te^{t} - 1$.

$$f'(t) = (t+1)e^{t}$$

Then

$$t_{n+1} = t_n - \frac{t_n e^{t_n} - 1}{(t_n+1)e^{t_n}} = t_n - \frac{t_n}{t_n+1} + \frac{1}{(t_n+1)e^{t_n}}$$

$$= \frac{t_n^2}{t_n+1} + \frac{1}{(t_n+1)e^{t_n}}$$

$$= \frac{1}{t_n+1}\left(t_n^2 + e^{-t_n}\right)$$

By looking at the plot of $f(t)$, one can pick $t_0 = 0$ as the initial point for the iteration.

After getting an appr. value for root t_*, we get an approximate root x_* by the relation $x_* = \frac{1}{2} t_*$. The appr. intersection point is

$$\left(x_*, e^{-x_*}\right).$$