Worksheet
11/04/2019

Name: _____

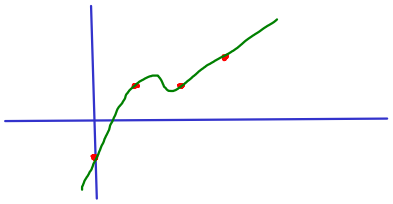
Find the polynomial P of degree ≤ 3 that interpolates the data (1,1), (2,1), (3,2), (0, -1). What is the value of $P(1.5)$?

$(x_1, y_1) = (1, 1)$

$(x_2, y_2) = (2, 1)$

$(x_3, y_3) = (3, 2)$

$(x_4, y_4) = (0, -1)$



$$L_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-2)(x-3)(x-0)}{(1-2)(1-3)(1-0)} = \frac{1}{2}x(x-2)(x-3).$$

$$L_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x-1)(x-3)(x-0)}{(2-1)(2-3)(2-0)} = -\frac{1}{2}x(x-1)(x-3).$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} = \frac{(x-1)(x-2)(x-0)}{(3-1)(3-2)(3-0)} = \frac{1}{6}x(x-1)(x-2).$$

$$L_4(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} = -\frac{1}{6}(x-1)(x-2)(x-3).$$

Then

$$P(x) = y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) + y_4 L_4(x)$$

$$= L_1(x) + L_2(x) + 2L_3(x) - L_4(x)$$

$$= \frac{1}{2}x(x-2)(x-3) - \frac{1}{2}x(x-1)(x-3) + \frac{1}{3}x(x-1)(x-2) + \frac{1}{6}(x-1)(x-2)(x-3)$$

After expanding and simplifying, we get

$$P(x) = \frac{1}{2}x^3 - \frac{5}{2}x^2 + 4x - 1$$