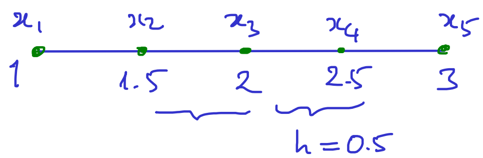
Worksheet

11/20/2019

Name: ______________________________

Approximate the integral $\int_1^3 x^2 dx$ by taking 5 equally spaced sample points $1 = x_1 < \ldots < x_5 = 3$ and using

- left-point rule,
- right-point rule,
- midpoint rule,
- trapezoidal rule.



$f(x) = x^2$

$y_1 = f(x_1) = f(1) = 1^2 = 1,$

$y_2 = 1.5^2 = 2.25,$

$y_3 = 2^2 = 4,$

$y_4 = 2.5^2 = 6.25,$

$y_5 = 3^2 = 9.$

* Left point rule:

$$\int_1^3 x^2 dx \approx h(y_1 + \cdots + y_4) = 0.5(1 + 2.25 + 4 + 6.25)$$
$$= \cdots$$

* Right-point rule:

$$\int_1^3 x^2 dx \approx h(y_2 + \cdots + y_4) = 0.5(2.25 + 4 + 6.25 + 9)$$
$$= \cdots$$

* Midpoint rule:

$$\int_1^3 x^2 dx \approx h\left(\frac{x_1+x_2}{2}\right)^2 + h\left(\frac{x_2+x_3}{2}\right)^2 + \cdots + h\left(\frac{x_4+x_5}{2}\right)^2$$
$$= 0.5\left(\frac{1+1.5}{2}\right)^2 + \cdots + 0.5\left(\frac{2.5+3}{2}\right)^2 = \cdots$$

* Trapezoid rule:

$$\int_1^3 x^2 dx \approx h(y_1 + 2y_2 + \ldots + 2y_4 + y_5) = 0.5(1 + 2(2.25) + \cdots + 2(6.25) + 9)$$
$$= \cdots$$