Worksheet

11/22/2019

Name: ______________________________

Let us compute approximately the integral $I = \int_1^3 x^2 dx$ by

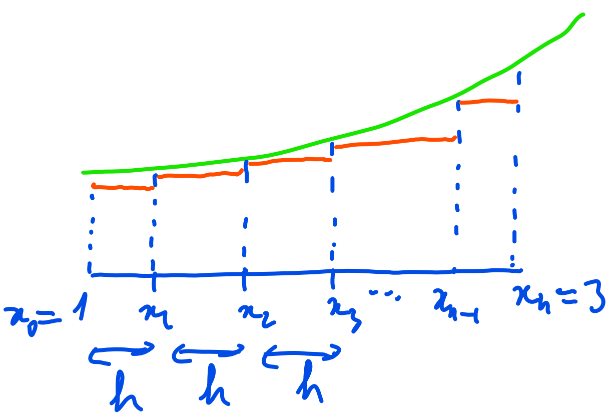
- left-point rule (call the sum L_n),
- trapezoidal rule (call the sum T_n),

with $n+1$ equally spaced sample points $1 = x_0 < \ldots < x_n = 3$.

(a) Write L_n and T_n using sigma notation.

(b) Find n such that L_n approximates I with error not exceeding $\epsilon = 10^{-4}$.

(c) The same question as in Part (b) for T_n.



$$\left.\begin{aligned} x_0 &= 1 \\ x_1 &= 1+h \\ x_2 &= 1+2h \\ &\ldots \\ x_k &= 1+kh \\ &\ldots \\ x_n &= 1+nh \end{aligned}\right\}$$ sample points, where $h = \frac{3-1}{n} = \frac{2}{n}$.

Left-point rule:

$$I = \int_1^2 \underbrace{x^2}_{f(x)} dx = \sum_{k=0}^{n-1} \int_{x_k}^{x_{k+1}} f(x)\,dx \quad \text{(summing over the subintervals)}$$

$$\approx \sum_{k=0}^{n-1} h f(x_k) \quad \text{(left-point rule)}$$

$$= \sum_{k=0}^{n-1} \frac{2}{n} x_k^2 = \frac{2}{n} \sum_{k=0}^{n-1} x_k^2$$

$$L_n = \frac{2}{n} \sum_{k=0}^{n-1} \left(1 + \frac{2k}{n}\right)^2$$

We know that

$$|I - L_n| \leq \frac{M}{2} \frac{(b-a)^2}{n} \quad \text{where} \begin{cases} a = 1, \ b = 3 \\ M = \max_{x \in [a,b]} |f'(x)| \end{cases}$$

We have $M = \max_{x \in [1,3]} |2x| = \max_{x \in [1,3]} 2x = 6.$

Thus, $|I - L_n| \leq \frac{6}{3} \frac{(3-1)^2}{n} = \frac{8}{n}$

To get $|I - L_n| < 10^{-4}$, we only need n such that

$$\frac{8}{n} < 10^{-4}.$$

This is equivalent to $n > 80000$. (A big number!)

The trapezoid rule will be mentioned in class soon.