Worksheet

11/27/2019

Name: ________________________________

1. Let us compute approximately the integral $I = \int_2^5 \frac{1}{x} dx$ by trapezoidal rule (call the sum T_n) with $n+1$ equally spaced sample points $2 = x_0 < \ldots < x_n = 5$.

   (a) Write T_n using sigma notation.

   (b) Find n such that T_n approximates I with error not exceeding $\epsilon = 10^{-4}$.

For Part (a), see lecture note

(b) We know that

$$|T_n - I| = e_n \leq \frac{\tilde{M}(b-a)^3}{12n^2}$$

where $\tilde{M} = \max_{[a,b]} |f''(x)|$,

$$[a,b] = [2,5],$$

$$f(x) = \frac{1}{x}.$$

We have $\tilde{M} = \max_{[2,5]} \frac{2}{x^3} = \frac{2}{2^3} = \frac{1}{4}.$

Thus, $e_n \leq \frac{1/4\,(5-2)^3}{12n^2} = \frac{9}{16n^2}.$

For $e_n \leq 10^{-4}$, we need $\frac{9}{16n^2} \leq 10^{-4}$.

Therefore, $n \geq 75$.

2. Approximate the integral in Problem 1 using Simpson's rule with $n = 6$. How large should n (even number) be such that the Simpson sum S_n approximates I with error not exceeding $\epsilon = 10^{-4}$?

Simpson's rule will be introduced
later in class.